Abstract

This article offers an analysis and mathematical modeling of the influence of one of the major factors of the World System macrodynamics throughout most part of its history (since the ‘urban revolution’) – the factor of interaction of civilizations with their barbarian periphery. The proposed mathematical model is intended to describe possible influence of interaction between civilizational core of the World System and its barbarian periphery on the formation of the specific curve of the world urbanization dynamics. It simulates completion of the phase transition, behavior of the system in the attraction basin and beginning of the phase transition to the attraction basin of the new attractor and is aimed to identify the role of the factor of interaction between the civilizational core and barbarian periphery in the formation of attractor effect during the completion of phase transition, that is for clarification of the reason why there was observed not only slowdown of growth rates of the main indicators of the World System development after completion of phase transitions during its development, but also their falling with the subsequent temporary stabilization near some equilibrium level. Achievements of modern barbarology, including the understanding of complexity of the barbarian periphery itself and its heterogeneity are considered. The basic principle of the proposed dynamic model is that sizes, power and level of complexity in realization of external policy functions in nomadic unions (empires) closely correspond to sizes, power and level of political culture and activity of the core states with which nomads constantly had to do (this point has been established in works of the known experts in nomadic studies). Various alternatives are shown in the model, when depending on power and size of one of the two components of the...
system 'civilization – barbarian periphery' studied by us, another one also changes significantly as it has to respond to the challenge properly, or can make less efforts feeling no threat or resistance. This principle is observed throughout the long period of the history of the World System. It is shown that interaction between the civilizational center and barbarian periphery really can explain some characteristic features of the World System dynamics in the 4th millennium BCE – the 2nd millennium CE. The ways of further development of the model are outlined.

**Keywords:** dynamic modeling, the World System, civilizations, barbarian periphery, urbanization, asabiyyah, technology, warfare.

The emergence and development of the world urban network was one of the main components of the World System's evolution that accelerated its development and increased its integration. Not without reason V. Gordon Childe focused on the urban revolution (Childe 1952: Chapter 7; Childe 1956). It is also quite clear that the processes of functional differentiation, social stratification and class formation proceeded in many ancient agricultural societies under a considerable influence of the ‘urban revolution’ (Alekshin 1986: 22). The city also implies a complex concentration of geographical, social, political, and sacerdotal resources and assets. ‘The city is a direct territorial concentration of multiple heterogeneous forms of human activities’ (Akhiezer 1995: 23). One can see the closest connection between urbanization, on the one hand, and the formation and development of civilizations and statehood, on the other.  

As is known, relatively large pioneer settlements (like Jericho in Palestine) vaguely resembling cities, emerged more than 9,000 years ago. In particular, about 7200 BCE Jericho was surrounded by a stone wall three meters thick, and four meters high (Lamberg-Karlovsky and Sablov 1992: 75). In the 7th – 6th millennia BCE, a number of settlements with estimated population of about 2,000 people appeared in Western Asia (Ain Ghazal, Beisamoun, Beida, Abu Hureira, Çatal Hüyük). However, it is undisputable that the first real cities appeared only in the 5th – 4th millennia BCE. And, finally, the first period of a rapid urban growth within the World System occurred between the second half of the 4th and the first half of the 3rd millennium BCE.

Our previous research (Korotayev 2006c, 2007a, 2007b; Korotayev, Komarova, and Khaltourina 2007: 169–177; Korotayev and Grinin 2012, 2013; Grinin 2017a, 2017b) has shown that the curve describing the dynamics of the world urban population has a rather peculiar form (see Figs 1 and 2).

---

1 See, e.g., Korotayev and Grinin 2006, 2007; Grinin 2007a; Masson 1989. This situation was typical for many regions: Ancient Greece (Gluskina 1983: 36; see also Frolov 1986: 44; Andreyev 1979: 20–21); Mesopotamia, in particular in the late 4th millennium and the 3rd millennium BCE (Dyakonov 2000: 46), a number of African regions; e.g., in South East Madagascar in the 17th century a few small Betsileo states originated in this way (Kottak 1980; Claessen 2000, 2002, 2004).
As can be seen, one can single out here three rather distinct periods of relatively fast world urban population growth: (A1) from the mid-4th millennium BCE to the mid-3rd millennium BCE, (A2) the 1st millennium BCE (roughly corresponding to the ‘Axial Age’); and (A3) the 19th – 21st centuries CE. Moreover, one can note two periods of relatively slow growth of the world urban population (including long phases when the urban population and the world urbanization level would hardly grow or could even considerably fall): (B1) from the mid-3rd millennium BCE to the late 2nd millennium BCE and (B2) between
the 1st and 18th centuries CE. Two other periods turn out to be essentially close to these epochs: Period (B0) immediately preceding the mid-4th millennium (when the world urban population did not grow simply because cities had not appeared yet and no cities existed on the Earth), and Period (B3) that is expected to begin in the 22nd century, when, according to forecasts, the world urban population will again stop growing in any significant way (since the World System urbanization is supposed to reach the saturation level along with the stabilization of the world population) (see, e.g., Korotayev, Malkov, and Khaltourina 2006a, 2006b, 2007; Korotayev, Komarova, and Khaltourina 2007; Korotayev 2008, 2009, 2012, 2013; Grinin 2006a, 2006b).

Note that the detected world urbanization dynamics correlates rather well with the dynamics of the World System political organization (Grinin and Korotayev 2006, 2007; Korotayev and Grinin 2007, 2012, 2013; Grinin 2016a, 2016b; Grinin, Ilyin, and Andreev 2016). Moreover, the above mentioned synchronous phase transitions to the new levels of the world urbanization and new complexity levels of the World System political organization temporally coincide with phase transitions to higher levels of the World System political centralization that were detected by Taagepera and that took place, according to his calculations, during periods A1, A2 and A3 (Taagepera 1997: 485).

Similar phase transitions appear to be observed with respect to the world literacy macrodynamics. In fact, during Period A1 we observe the emergence of the first literate people whose share in the world population by the end of this period had reached the level of decimals of a percent and fluctuated at this level during Period B1. And it is no coincidence. Although literate people could live not only in cities, nevertheless, their number in cities was incomparably larger than in rural areas. During Period A2, the world literacy rate grew by an order of magnitude and amounted to several percent of the world total population; it fluctuated at this level during Period B2 till the late 18th century when Period A3 started. During that period the world literacy level reached the level of dozens percent, and by the beginning of Period B3 (presumably in the 22nd century) it is likely to stabilize at the hundred-percent level (see, e.g., Korotayev 2006d; Korotayev, Malkov, and Khaltourina 2006a, 2006b).

In fact, the above-mentioned phase transitions can be regarded as different aspects of a series of unified phase transitions: Phase Transition A1 from medium complexity agrarian societies to complex agrarian ones, Phase Transition A2 from complex agrarian societies to supercomplex ones, and, finally, Phase Transition A3 from supercomplex agrarian societies to postindustrial ones (within this perspective, the period of industrial societies turns out to be a period of phase transition B2 – B3). These phase transitions are also exceptionally strongly connected with production revolutions and transitions from one principle of production to another (see in more detail: Grinin 2003a, 2007a, 2007b; Korotayev and Grinin 2006, 2007). The period of the first attrac-
tor (the first phase), in particular, is connected with the first variant of the intensive phase of agrarian revolution (transition to irrigation agriculture); the second attractor/phase – with the second variant of the intensive phase of agrarian revolution (transition to intensive plow non-irrigation agriculture). From the 16th to the first half of the 20th century (especially the 19th – the first half of the 20th century) the phase transition was connected with the transition to the industrial principle of production. The period from the end of the 20th century and (presumably) the whole 21st century is connected with the transition to scientific and information/cybernetic principle of production (for more details see Grinin 2006a, 2006b; Grinin L., Grinin A., and Korotayev 2017a, 2017b).

The proposed mathematical model aims at analyzing the possible impact of the interaction between the World System's civilizational core and its barbarian2 periphery on the formation of a specific curve of the world urbanization dynamics. It describes the completion of the phase transition, system's behavior in the basin of attraction, and the start of the phase transition to a new basin of attraction. This model also identifies the role of the interaction between civilizational core and barbarian periphery in the formation of the attractor effect during the completion of phase transition. In other words, we try to find out why after the phase transitions were completed there was observed not only the slow-down in the growth rates of the main indicators of the World System development but also their decline with subsequent temporary stabilization at some equilibrium level (let us note that the offered model cannot describe the fluctuations observed at the respective levels).

The issues of coexistence, interaction and struggle between civilization and barbarian periphery are extremely important for understanding of the evolution of the World System over the last five thousand years after the emergence of the first states and civilizations. This also remained relevant up to some extent for the Modern Period up until very recent times. In some regions, like, for example, the Middle East and North Africa, the non-state tribal and chiefdom forms of political organization continued coexisting, competing or cooperating in the military field with the states up to the early 20th century. In particular, even in the territory of Egypt, one of the most ancient civilizations, in the second half of the 18th century the Bedouin raids were a great threat for sedentary populations (as evidenced by the famous Egyptian historical chronicler of the late 18th and the early 19th centuries Abd ar-Rahman al-Jabarti [1978]). In the middle of the 20th century within the territory of another most ancient, Chinese, civilization one could find a true internal barbarian periphery which made regu-

2 Needless to say that, following Lewis H. Morgan (1877) tradition, we use the term ‘barbarian’ as a purely technical one – devoid of any pejorative implications (note that Morgan himself in no way despised the ‘barbarians’ – he – in company with Friedrich Engels [1884/1978] – rather admired them).
lar incursions into the center. One can recollect a similar situation in the Caucasus in the 19th and even in the 20th centuries.

As is known, there are great differences in the definition of civilization (see, e.g., Grinin 1997, 1998a, 1998b, 1998c; Grinin and Korotayev 2008: Introduction). In this paper we operationalize civilization or civilizational core (center) of the World System as the societies of the World System core with urban settlements; while the peripheral communities without urban settlements are defined as ‘barbarian’. Within the framework of the present mathematical model the existence of cities is assumed to be the only formal characteristic of civilization.

As Vera P. Budanova (2002: 168) notes, there are some directions in modern barbarology which study general relations between barbarian world and civilization (see, e.g., Budanova 1990, 1994, 2000, 2002; Masson 1986, 1989; Barfield 2006, 1991; Kradin 1992, 2001a, 2001b; Kradin and Bondarenko 2002; Pershits and Khazanov 1978; Khazanov 1975, 2002, 2006; Sannikov 2002, 2003, 2005; Kradin, Bondarenko, and Barfield 2003; Grinin and Korotayev 2013, 2014, 2018). Nevertheless, it is a very broad subject, and many of its aspects are hardly well scrutinized. Still, the following conclusions drawn by barbarologists are especially important for our subject: 1) the center and barbarian periphery are considered as closely related elements of a single pan-

---

3 E.g., the Yi people (or Nuosu people) in the high mountain region of Liangshan of China’s Sichuan province. There were four ‘classes’ in this society, one of which (actually [Nuosu] – ‘black’) contrary to subordinate ‘whites’ was superior, noble, and therefore it did not participate in productive labor. The rest three classes were in different degree of dependence – from semi-bond to slavish against the background of the absence of any developed political structure (Its and Yakovlev 1967; Kubbel’ 1988: 241–242). Such a situation developed from the 7th – 9th centuries CE after the cattle breeding tribes had subordinated farmers (Its and Yakovlev 1967: 79). Slavery was widespread in this society. Herewith the Nuosu often attacked and captured the Han people, reducing them to slavery. Thus, in 1919 the Liangshan Yi people captured and took away more than 10,000 people from neighboring counties to their highlands. In the early 19th century the total population of the Liangshan Yi was rather small, numbering about 10,000 people. But in 1838 it amounted already 40–50,000, and in 1910 – about 200–300,000. It continued to increase, having reached 630,000 people in the mid-1950s, among whom non-assimilated Han slaves totaled 50,000–60,000 (Ibid.: 79–80).

4 E.g., the General Anton Denkin in his History of the Civil Strife in Russia (Denkin 1993: 122) speaks about the Ingush as the most organized among the Caucasian peoples who took advantage of the anarchy during the Civil War and systematically plundered and terrorized all the neighbors. In fact, the most recent events in the Caucasus in Russia still remind us ‘living vestiges’ of such relationships between civilization and barbarian peripheries.

5 The identification of this characteristic as a working criterion of civilization within our mathematical model should, of course, be treated just as an assumption and is explained by the necessity to determine in an operationalizable way within the present formal/mathematical research the societies forming the World-System core and having urban settlements as distinct from peripheral societies (designated here as ‘barbarous’) lacking those settlements. Let us note that it does not contradict some researchers’ rather fair statements that in the context of their research this characteristic can be substituted, e.g., by the presence of monumental buildings (see, e.g., Masson 1989).
Oecumene system (= the World System) in which peoples with different levels of socio-cultural complexity interact (see, e.g., Pershits and Khazanov 1978: 4; Budanova 2002: 168); 2) within the barbarian periphery itself a certain center – a ‘core’ of the barbarian world – can be formed in relation to civilization, which in many respects defines relationships between civilization and barbarians and with that part of barbarians who inhabited territories that were distant from the civilization core (the emergence of such centers, as a result of the civilizational core pressure quite often led to the growth of collective solidarity [asabiyyah] of barbarians that we try to account for in our model); 3) the complexity level of the barbarian alliances (especially among the nomads) closely corresponds to the size and level of political culture of states with which they were in contact (see, e.g., Barfield 2006); and 4) foreign policy and economic (trade) interests played a significant role in the relations between barbarian world and civilization, however, military contacts prevailed there (Budanova 2000, 2002; Kradin 1992, 2001a; Barfield 1991, 2006).

Elsewhere we have already analyzed the possible role of interaction between the civilizational core and barbarian periphery (see Korotayev, Malkov, and Khatourina 2007: 189–208; Grinin 2003b, 2004a, 2004b, 2011; Grinin and Korotayev 2013, 2014, 2018; Grinin et al. 2004, 2006) and considered the reasons of an essential decline (up to negative values) of the growth rates of the main indicators of the World System development in the 1st millennium CE after the completion of A2 phase transition to supercomplex agrarian societies (Korotayev, Malkov, and Khatourina 2006b; Zinkina, Ilyin, and Korotayev 2017). Note that the above-mentioned analysis allowed identifying that factor as one of very important causes (but not the only one) of the considered phenomenon. Thus, we have come to the following preliminary conclusions:

The fact that the regime of hyperbolic growth changed after the World System's political centralization had reached critically high level of hyperbolic rates (in the early 1st millennium CE the absolute majority of World System's inhabitants turned out to be under control of only four empires – Roman, Parthian, Kushan and Han) is not accidental also for some other reasons. The rapid growth of political centralization in the 1st millennium BCE was driven by the diffusion of iron metallurgy (for more details see Grinin and Korotayev 2008: Ch. 6; Korotayev and Zinkina 2017; Zinkina, Ilyin, and Korotayev 2017), which not only considerably increased the Earth's carrying capacity, but also led to the development of production of rather cheap and effective weapons which promoted the formation of numerous armies without which the emergence of the world empires would be almost impossible. However, this process had important side effects. The politically centralized systems

---

6 At the same time in those barbarian polities such leaders would constantly change (Budanova 2002).
Andrey V. Korotayev and Leonid E. Grinin

quite often secure military superiority through the development of specialized military subsystems — rather small but well trained and professional armies. However, to preserve this superiority there is necessary to have monopoly on certain effective types of weapons (war chariots, bronze weapons, etc.). If the revolution in production of means of violence takes place and the monopoly on them cannot be efficiently supported (e.g., in case of emergence of iron weapons), the less politically centralized societies with a high proportion of military active population get considerable advantage and in military terms can become stronger than politically centralized societies. This was the case in many parts of Oecumene of the Old World in late antiquity. Moreover, less politically centralized societies with a greater share of military active population could considerably increase their military efficiency without noticeable increase in their political centralization or internal differentiation, for example, through nomadization, growth of specialization on cattle breeding since the herder’s everyday work and the character of his socialization make him a combat-effective warrior. Nomadic cattle breeding with a widespread use of herders-riders could considerably increase military potential of such societies without additional political centralization and functional differentiation. In this context it is important for us that the side effect of the technological shifts of the first millennium BCE was strengthening of the barbarian periphery's military potential in general and nomadic socio-political systems, in particular... As a result, the nomads got a consistent military superiority over the settled societies throughout most part of the ‘Junior Hyperbole’ epoch (additionally strengthened by the invention and diffusion of stirrups and sabers); this led to an additional reduction in the World System's demographic growth rates not only due to mass depopulations resulting from recurring nomadic invasions, but also as a result of some decrease in the Earth’s carrying capacity in many important zones of the World System due to the pressure of barbarian (and, in particular, nomadic) peripheries (here we could recollect the Russian ‘bread-basket’ – Black Earth region which through the most part of the 2nd millennium was known as the Wild Field since the lands in this region were almost not cultivated because of the threat of nomadic raids) (Korotayev, Malkov, and Khaltourina 2007: 207–208).

Let us note that a systematic military superiority at a certain phase of the World System evolution does not mean a constant superiority. China, for example, defeated the Xiongnu a few times, carrying out deep raids in their lands (see, e.g., Gumilev 1993; Kradin 2001a), similar as the Russian dukes did with respect to the lands of Cumans (see, e.g., Rybakov 1966a: 561–562). Therefore, we actually deal with an unsteady balance of forces between the barbarian periphery and civilizational center but this balance could change under some circumstances. In a certain situation, the barbarian periphery’s pressure would come over civilization or, vice versa, civilization would invade the barbarian
A Model of Center-Periphery Interaction

periphery. Thus, one can speak about certain cycles when phases of civilizational center's expansion and the barbarian periphery retreat are alternated by the phases of barbarian periphery expansion and civilizational center's retreat.

It is worth mentioning that depending on the power and scale of one of the two components of ‘civilization – barbarian periphery’ system, the other element would also significantly change to give an adequate response to the amplified (or changing in some other way) challenge; otherwise, feeling no threat or resistance it can make less efforts. Anyway, it was noticed, that in nomadic alliances (empires) the size, power and complexity level of foreign policy functions correlated closely with the size, power and level of political culture and activity of the states with which the nomads constantly interacted (see, e.g., Barfield 2006: 429). This can also explain the situation described below in the model, when a civilization expands to the barbarian periphery rather actively, while the latter is unable to actively resist the former. It may happen because the barbarian periphery turns unable to adapt to the power and size of the advancing civilization yet. After absorbing the part of the barbarian periphery which is less capable to resist, especially the territories with environment, suitable for the civilization's economic expansion (and with peoples who are somehow ready to become a part of civilization), civilization can face more persistent representatives of the periphery especially those living under marginal conditions. As a result, the above-mentioned dynamic equilibrium can be established for certain periods (sometimes for a rather long time).

Although a rather long coexistence of civilization and barbarian periphery is obvious, each part of this dynamic system tries to weaken or even destroy the other at every opportunity, so there emerges a situation of ‘interdiffusion’ when various innovations are borrowed (mostly by the barbarians from civilization, but sometimes vice versa) and also civilization uses the barbarians for its own needs. As a result one can observe an accelerated development of the barbarian periphery which in order to have advantages and resist civilization tries to develop similar political and social forms. This usually aims primarily at achieving a military balance or military superiority, and also at achieving the parity of prestige. This also involves ideology which can be rather developed among the barbarians. The latter is important for understanding of the asabiyah concept which was developed by an outstanding medieval Arab thinker Abd ar-Rahman Ibn Khaldūn as a scientific category (Ibn Khaldūn 1958, 2004) and introduced into the scientific vocabulary of modern Cliodynamics by Peter V. Turchin (2003, 2007) who, in our opinion, quite reasonably interprets this concept as ‘collective solidarity’.

It is a peculiar ideology of tribal solidarity, which allows uniting barbarian people into powerful military force, for example, when putting together groups of tribes. Therefore, Morton Fried (1967) has reason to state that tribes are the secondary non-primitive formations emerging under the influence of neigh-
boring communities with significantly higher level of sociocultural complexity (see also Korotayev 1997a, 1997b, 1998, 2000a, 2000b, 2003, 2004, 2006b, 2006c; Grinin 2007a). In fact, many analogue forms of polities of the ‘main sequence’ are often secondary phenomena associated with the impact of civilizational center, or their development is significantly modified being effected by more developed neighbors (see Grinin 2007a). For example, such a modification might have happened in the development of the Scythian polity since the Scythians actively communicated with the Medians and Persians, and later with Greeks (see, e.g., Dyakonov 1956; Khazanov 1975). Moreover, such forms quite often emerge just because they best fit the marginal environment, while civilizations, as a rule, emerge in the environment more favorable for the development of intensive production. It is natural that the analogues in barbarian periphery possessing certain environmental, economic and demographic features could get along without cities. Only some barbarians had a developed system of cities as it was in Gaul where there were up to a thousand of ‘genuine cities’, and in some of them population reached tens thousands people (Shkunaev 1989: 134, 143). The size of some cities was 100 and more hectares, and they were secured by powerful walls (see Filip 1961: 116–129; Mongait 1974: 248–253).

Thus, due to interaction of different kind between civilization and barbarian periphery: a) the World System expanded and became more and more complex⁷; b) the socio-political, economic and cultural level of the barbarian periphery generally increased; c) the civilizational level, including urbanization, could temporarily decrease due to generally increasing size of the World System, and temporal ‘barbarization’ of extensive territories as it was repeatedly observed in the 1st millennium CE (especially in Europe).⁸ One can apply here Adolf Leo Oppenheim’s idea (1990: 88) about constant counteraction between anti- and pro-urbanistic trends in ancient Mesopotamia and in the ancient world in general, while the barbarian periphery was the most important agent of the former trend.

It is also worth mentioning that when speaking about each certain barbarian onslaught towards the civilizational zone, as well as about definite periods of such mass movement, we can hardly know the exact reasons that launched such migrations. For example, Budanova (2000: 5–6) writes that there is still no definite answer to the question what triggered the migration engaging territories

---

⁷ This may have involved the integration of a number of civilized societies by barbarian conquerors. We think that the Mongolian amalgamation is one of the most significant in this context since for a certain period it strengthened the relations within the World System from the Pacific Ocean to the Atlantic (see, e.g., Abu-Lughod 1989, 1990).

⁸ One can also assume that the World System expansion rate could be sometimes inversely proportional to the rate of quality growth of its particular parts and processes (such as urbanization).
from Scandza to Mauritania, from China to the Pyrenees in the Great Migration Period (the 3rd–7th centuries CE).9


In the proposed model the World System is assumed to be divided into three main geographical zones: (1) small (1 mln km²) and highly productive zone; (2) a larger size zone with average producing capacity (24 mln km²) which surrounds Zone 1; and (3) the largest in size (96 mln km²) and the least productive zone surrounding Zone 2 (see Fig. 3).

![Fig. 3. Spatial structure of the World System assumed in the model](image)

It can be assumed that the first cities originated in Zone 1 (see, e.g., Korotayev and Grinin 2006, 2012, 2013; Grinin and Korotayev 2009a, 2009b) which therefore, can be identified as the ‘civilizational center’. It is assumed that the initial level of technological development in this center \(T_{c0}\) is significantly higher than that for the barbarian periphery \(T_{b0}\) coinciding with Zone 2 at the start of computer simulation. At this point, Zone 3 with the lowest initial level of technological development \(T_{h0}\) is considered as the World System's hinterland.

At the first stage of computer simulation the model's major scenario describes the initial vigorous territorial expansion of the civilizational center supported by its more developed technologies, which in combination with signifi-

---

9 However, the possible general reasons of the barbarian periphery pressure on civilizational center are quite clear: the demographic pressure, the shortage of resources (land, pastures) correlated with this and other (natural in the first turn) factors; aspiration to the spoils of war; and pressure of enemies (i.e., conflicts within the barbarian periphery) and other similar factors.
cantly denser population of the civilization zone results in a significantly higher military potential. In the proposed model, the civilization's territorial dynamics is mathematically described by means of the following differential equation:

\[
\frac{dA_c}{dt} = a(M_c - M_b),
\]

(Eq. 1)

where \( A_c \) is the territory controlled by the civilizational core; \( M_c \) is the military potential of the civilizational core; \( M_b \) is the military potential of the barbarian periphery; \( a \) is the constant which determines the rate of transformation of military superiority into territorial acquisitions (the calculation pattern for \( M_c \) and \( M_b \) values will be described below, see Eqs 2 and 3).10

However, after a while this expansion is exhausted in the major scenario of the model and the barbarian periphery's counterattack unfolds.11 Note that in the suggested model (as well as in historical reality) less numerous and technologically backward barbarians can put pressure on more numerous and technologically advanced ‘civilized’ enemies. This effect may be produced by the following factors:

1) A higher military participation ratio that was characteristic of the barbarians. It is proved by written, ethnographic and even archaeological sources. For example, in some territories occupied by the German tribes before the Great Migration epoch about 80% of males were buried with iron weapons (see Gurevich 1999: 44). One should also mention the early military training for boys among many barbarian (especially nomadic) peoples, for example, among Huns, Mongols or Turks when they were nomads (see, e.g., Nefedov 2008).12

10 In real history, it could be just the result of demographic pressure of migrants who would absorb numerically insignificant aboriginals, or the result of combination of demographic dissipation and military superiority. Thus, many barbarians just disappeared as ethno-social entities and were assimilated by civilized peoples. But those who survived became ethnicities of a new generation capable of both military and cultural opposition, and development of their own complex political systems which led to creation of analogues of the state among barbarians (see, e.g., Grinin 2001–2006, 2003b, 2004b, 2007a).

11 In real history it was most often observed that civilization reached the limits of the natural zones, suitable for its economic pattern (and apparently, it would be worth taking into account this effect in future mathematical models). Note, also, that ‘barbarous counterattack’ in reality might start both after the period of established power balance, and sometimes at once without intermediate period of balance. Khan Konchak’s campaign against the Russians in 1185 following Duke Igor Novgorod-Seversky’s unsuccessful campaign on the Cumans can serve a classic example here. Moreover, the Cuman troops moved toward Rus’in three directions: toward deserted Igor and Vsevolod’s principalities, toward Pereyaslav and Kiev, ‘where Konchak was attracted by the memories of Khan Bonyak knocking with sabre on Kiev’s Golden Gate’ (Rybakov 1966b: 595).

12 ‘Training of the Turkish archers does not appear to have been inferior to Mongolian archers, and similar to the Mongols, constant trainings also promoted the development of muscles of arms. ‘From eight, or even seven years old they began to shoot at a target, – the imperial ambassador Ghiselin de Bousbecq wrote, – and for ten or twelve years they would practice in archery. This continuous training strengthened muscles of their arms and gave them such a skill that they could hit the smallest targets with their arrows’ (Nefedov 2008).
The most important role of this factor in explaining the cases of successful advance of the barbarian periphery on civilizational center was described in the 16th century by the Ethiopian monk Bahrey in his well-known History of the Galla. Bahrey tried to explain why the politically centralized Ethiopian state was constantly defeated by the politically less centralized and less developed Galla (Oromo) tribes ("How is it that the Galla defeat us though we are numerous and well supplied with arms?" Bahrey 1976 [1593]: 140). The answer which Bahrey proposed is very interesting and convincing: just because the Ethiopian society was much more developed and socially differentiated (i.e., actually more ‘civilized’), it suffered continuous defeats in the fight against less developed ‘barbarians’, the Galla. In this case the high level of internal differentiation (‘civilization’) becomes a source of military weakness:

How is it that the Galla defeat us, though we are numerous and well supplied with arms?... It is because our nation is divided into ten classes, nine of which take no part whatever in war, and make no shame of displaying their fear; only the tenth class makes war and fights to the best of its ability. Now, although we are numerous, those who can fight in war are few in number, and there are many who do not go to war. Of these classes, the first is that of the monks, of whom there are vast numbers. Among them are those who become monks at an early age, drawn thereto by the other monks while they are studying, as indeed was the case with him who has written this history, and others like him. There are also others who become monks because they fear war. A second group is composed of those who are called dabtara, or clerks; they study the holy books and all works relating to the occupations of the clergy; they clap their hands and stamp their feet during divine service, and have no shame for their fear of going to the wars. These people take as their models the levites and priests, namely, the sons of Aaron. The third group is that of the people called Jan Hasana and Jan Maasare, who look after the administration of justice, and keep themselves from war. The fourth group is formed by those who escort the wives of dignitaries and the princesses; they are vigorous, brave, and strong men who nevertheless do not go to war, for they say, ‘We are the protectors of the women’. The fifth group calls itself ema gelle, ‘elders’; they are the lords and hereditary landowners: they share their land with their laborers, and are not ashamed of their fear. The sixth group is that of the laborers in agriculture, who live in the fields and have no thought of taking part in war. The seventh group is composed of those who engage in trade and gain profit thereby. The eighth group is that of the artisans, such as the smiths, scribes, carpenters, and such-like, who know not the art of war. The ninth group is that of the wandering singers, those who play the qanda kabaro [a small drum] and the bagana, whose profession is to beg, to collect money. They invoke blessings on those who reward them, flattering them with vain praises and idle panegyrics; while those who refuse to give them presents they curse, though they are not blamewor-
thy for this, for, as they say, ‘This is our custom’. Such people keep themselves as far as possible from war. The tenth group, finally, is composed of those who carry the shield and spear, who can fight, and who follow the steps of their king to war. It is because these are so few in number that our country is ruined. Among the Galla, on the contrary, these nine classes which we have mentioned do not exist; all men, from small to great, are instructed in warfare, and for this reason they ruin and kill us (Bahrey 1976 [1593]: 140–141).13

In our model the higher coefficient of military participation that was typical for barbarians is mathematically described in Eq. 3 by giving b coefficient (representing the military participation ratio of barbarians here, i.e., the percentage of barbarian population participating in military operations) significantly higher value than that of c coefficient (representing the military participation ratio for the ‘civilized’ population) in Eq. 2. For example, in the computer si-

13 However, it is worth noticing that military forces of small polities were quite often comparable with military forces of large ones in those political entities of civilizations where the level of military participation of inhabitants was high (e.g., in some civil communities, in particular in Greek poleis, Roman civitas, some medieval cities). The best known example is Greek-Persian wars when an alliance of civil communities with a high military participation ratio defeated a low military participation ratio empire. Note also that the suggested mathematical model does not consider the following factors of barbarians’ military superiority (which it would be worth considering in future generations of similar mathematical models): a) a high mobility of some barbarian peoples in comparison with settled farmers which is quite often defined by their own way of life, as well as a low specific useful biomass output per unit of economically exploited territory (that causes the need of moving in order to increase their zone of economic exploitation). Especially it refers to nomads and the sea peoples, as well as the inhabitants of those places where rivers constituted the main communication lines. From the very beginning water transport was the main means of long distance connections (McNeill 1995). Therefore we suggested an idea that it is necessary to multiply the number of inhabitants among herders and seamen by the coefficient of their mobility for considering their potential for the intensification of political complexity growth processes (Korotayev 1991; Grinin 2007a). Such mobility often secures possibilities of the rapid advance in huge territories where civilizations are located; b) Higher prestige (concerning the whole population) of military activities. In other words, in a number of civilizations military professionals had no such prestige as priests or officials. For example, the founders of the Song Dynasty in China (960–1279) significantly downgraded and changed the position of military elite in order to prevent the possibility of the ‘military coups’ that undermined the stability of the political system of their predecessors (Wright 2001). But even where military estate stood high (as, e.g., in medieval Europe or Japan), monopolization of military affairs in their hands led to the fact that most of population were specialized in peaceful occupations and as a result their military potential was close to zero. So, for example, in the 8th and 9th centuries in Charlemagne’s empire, especially in France, the hardships that came along with military service actively induced peasants to pass voluntarily under the protection of large secular and religious landowners, thereby even by sacrificing their civil freedom (see, e.g., Gurevich 1970: 145–183). In Russia, it was not uncommon when voluntary transfer of noble children to serf status took place in order to save them from military service. At the same time participation in military affairs was very honorable among barbarians (especially in state analogues), and very often volunteers’ participation was sufficient to support major military actions in such affairs (see, e.g., Fenton 1978: 127 on the Iroquois).
mulations whose results are given below for the main scenario of our model, the value of \( c \) is 0.05, whereas the value of \( b \) coefficient is 0.2.

\[
M_c = cN_cT_cH_c, \tag{Eq. 2}
\]

where \( N_c \) is the size of ‘civilized’ population; \( T_c \) is the level of technological development of the civilizational core (for simplicity it is assumed that the level of development of military technologies of civilization is proportional to the general level of its technological development; therefore, within this model it is not identified as a separate variable); \( H_c \) is the level of asabiyyah of civilized population (we will dwell upon this variable below);

\[
M_b = bN_bT_{ab}H_b, \tag{Eq. 3}
\]

where \( N_b \) is the number of inhabitants of the barbarian periphery; \( T_{ab} \) is the level of development of military technologies in the barbarian periphery (it is assumed that this variable is not identical with the general level of technological development of the barbarian periphery; the implications of this assumption will be considered below); and \( H_b \) is the level of barbarians’ asabiyyah.

2) Borrowing of military technologies by barbarians happened at higher rates, than borrowing of non-military technologies (the chosen by us way to model this assumption mathematically will be described below). For example, due to the fact that Mongols borrowed siege equipment and technology from China, they were able to take a lot of cities successfully. Thus, in this case borrowing of military innovations was one of important reasons of mass destruction of cities of the World System in the 13th century. Nevertheless, the abovementioned facts about borrowings refer not only to weapons, but also to the strategy, tactics, and organization of the army. Quite often barbarians just imitated the structure of armies (or separate military institutes) of the neighboring civilizations. For example, the German leader Marobod (the late 1st century BCE — the early 1st century CE), having united Marcomanni with the Lugians, the Mugilones, Ghots and other Germanic tribes, created a large army on the Roman pattern which numbered 70,000 of infantry and 4,000 of cavalry (SIE 1966: 123).14

14 By the way, such borrowings became the main impulse for transformation of the non-state systems into the early state society quite often. This happened, e.g., as a result of the borrowing of iron weapons, and later fire arms (in particular, one could see examples of the latter in Madagascar in the 17th century [Deshan 1984: 353; Ratzel 1902, vol. 1: 445], or in Tahiti and Hawaii in the 18th century [Service 1975; Earle 2002: 86; for more details about similar cases see Grinin 2007a]). Note some more points here which are not considered in the present version of our model, but which it would be worth considering in the next generation of the models of interaction between the civilizational center and barbarian periphery. A) Barbarians themselves could be inventors of important military innovations. It is fair enough since many people considered war as the most important issue and became professionals of military attacks and robberies. Sometimes such inventions helped some barbarian chieftoms to defeat others. A classic example
3) The beginning of forceful expansion of the civilizational core upon the barbarian periphery can be interpreted as the formation of a metaethnic border between the civilization and the barbarian world. As was clearly demonstrated by Peter V. Turchin (2003, 2005, 2007), the formation of such a metaethnic border tends to lead to a significant increase in collective solidarity (asabiyyah) in that party that turned out to be under pressure.\footnote{Those, who withstood it, found adequate responses to the challenge that finally led to the selection of types of barbarian communities most adapted to the fighting against civilization.} As a result, if at the beginning of its forceful expansion civilization faced scattered groups of barbarians incapable to produce any effective resistance, further on these groups began to cooperate more and more among themselves for putting up resistance, and civilization had to deal with more and more united and large coalitions of barbarians (which were formed in many respects as a reaction to forceful expansion and were able to show more and more effective resistance, and further to start was that Shaka, the leader of the Zulu people, who applied a new type of cold weapon that in many respects promoted progress of his army and formation of the empire (Ritter 1968; Ratzel 1902, 2: 116). As a result, the Zulu polity was transformed from the pre-state level to the one of the state. There were cases when such inventions promoted expansion of barbarians against civilization. East Germans in the 5th century CE probably invented some kind of huge backsword with the straight sharpened blade (scramasax) which was up to 80 cm long. It was a typical saber weapon capable of giving terrible wounds which increased the power of a horse soldier. Therefore, it was borrowed by Huns, and then Goths and Francs (Kardini 1987: 263–264). The striking example of such innovations of barbarians were ships and naval tactics of Vikings who ‘were second to none at sea’ and whose sea advantage was often absolute (Gurevich 2005: 41 and ff.). One can also mention, e.g., military tactical and organization innovations used in Genghis Khan’s army, which undoubtedly played a great role in the Mongol victories. Thus, if barbarians and civilization were incomparable by cultural level, they could be quite comparable as regards their military-strategic levels, and quite often barbarians also had superiority, but at the same time kept such forms of organization of society which, according to to the well-known expert of nomadic studies William Irons, were real alternatives to state organization (Irons 2002, 2004) and could reproduce themselves without cities (though they could control cities populated by conquered peoples). B) In the process of weakening collective solidarity (– asabiyyah) of civilizations and states, conflicting parts of civilization begin to use barbarians as allies, which gives them a chance to interfere with affairs of the civilization core. One can recollect that the author of The Song of Igor’s Campaign wrote that dukes began ‘to forge feuds for themselves’, and ‘to draw the pagans onto the Russian land’. The late Roman and Byzantine history gives a lot of examples of the ‘integration’ of barbarians into policy of civilization. A classic example is a tragedy of post-Roman Britain. After the withdrawal of the Roman troops from Britain in 410 CE, the Britons (Romanized British Celts) searching for the defenders from attacks of the Irish and Scottish barbarians invited the Saxones and gave them some land (thereby having exercised a certain social innovation, which was, however, repeatedly used in the Roman world with its practice of ‘fighting against barbarians with barbarians’ hands’). But having seen weakness of the Britons, the Saxones ceased to obey the local authorities and together with the Angles and the Jutes became eventually the owners of the country. And, despite their prolong and persistent resistance, the Britons were partly expelled, partly enslaved, and partly destroyed. Therefore, Anglo-Saxon barbarous kingdoms emerged in Britain in place of the ‘Briton’ state (e.g., Blair 1966: 149–168; Chadwick 1987: 71). Thus, military opportunities of barbarians could significantly increase with their involvement into military-political affairs of civilization.
successful counterattacks). As has already been mentioned above, Turchin suggests using for denoting ‘collective solidarity’ the term *asabiyyah* that was introduced into the scientific discourse by Abd ar-Rahman Ibn Khaldun\(^\text{16}\) (1332–1406).

In the model, the dynamics of barbarian *asabiyyah* \(H_b\) is described mathematically by means of the following equation:

\[
\frac{dH_b}{dt} = e \times \frac{dA_c}{dt},
\]

(Eq. 4)

where \(e\) is a constant. It means that the higher the rates of forceful territorial expansion of civilization, the higher the growth rates of barbarians’ *asabiyyah*\(^\text{17}\).

Respectively:

\[
\frac{dH_c}{dt} = -e \times \frac{dA_c}{dt},
\]

(Eq. 5)

where \(H_c\) is *asabiyyah* of civilized population.

Note that it means that *asabiyya* of civilization begins to grow under the pressure of barbarians, and the stronger this pressure is, the quicker it grows (for more details see Turchin 2005).

While describing population dynamics, we base ourselves upon the simplified version of the compact model of demographic, technological and economic development of the World System (Kremer 1993; Korotayev 2005, 2006d, 2007, 2008, 2009, 2012, 2013; Korotayev, Malkov, and Khaltourina 2006a, 2006b, 2007; Korotayev and Malkov 2012; Zinkina, Malkov, and Korotayev 2014; Korotayev and Malkov 2016; Korotayev and Zinkina 2017; Grinin 2003a, 2012; Grinin L. and Grinin A. 2015, 2016; Grinin A. and Grinin L. 2015; Grinin and Korotayev 2016; Grinin L., Grinin A., and Korotayev 2017a). We make a Malthusian assumption that throughout the most part of the period of existence of the humankind, the human population was limited by the level of development of life-supporting technologies. As in simplified Kremer’s model (Kremer 1993: 685), we assume that population comes to technologically determined level of the Earth’s carrying capacity instantly (or, in other words, instantly fills the ecological niche expanded as a result of technological growth).\(^\text{18}\) Besides, we take into account the fact that territory with a higher


\(^{17}\) We also assume that with the increase in barbarians’ asabiyyah the rate of borrowing of military technologies of civilization increases (this assumption is modeled by the Eq. (8’’’)). We also assume that variable \(H\) cannot have negative values.

\(^{18}\) Let us note that it deprives us of an opportunity to describe cyclical dynamics of the system in the basin of attraction (see, e.g., Korotayev, Komarova, and Khaltourina 2007) that would bring dynamics generated by the model considerably closer to actually observable one, but at the same
natural productivity can support the existence of a larger population at the same level of technological development, than the territory with smaller natural producing capacity, and otherwise under equal conditions a larger territory can support a larger population than a smaller territory. Thus, the size of population \( N \) of some zone with productivity \( F \) and area \( A \) at the level of development of life-supporting technologies \( T \) will be described mathematically by means of the following equation:

\[
N = gFTA, \quad (\text{Eq. 6})
\]

where \( g \) is a constant.

As a result, the mathematical description of the population for year \( i \) for a hinterland of the World System (Zone 3) appears to be the simplest one in our model, since we have initially assumed that the territory occupied by it throughout the modeled period remains constant, and the level of technological development is the same for the whole zone:

\[
N_{3i} = gF_3T_3A_3. \quad (\text{Eq. 7})
\]

The situation with the civilizational core and barbarian periphery of the World System is a little more complicated. The matter is that throughout the most part of the modeled period the civilization zone is divided into two subzones with different natural productivity, i.e. the core of the civilization zone with high natural productivity (~ Zone 1) and the periphery of the zone corresponding to the part of less productive Zone 2 taken by the civilization from ‘barbarians’. Thus,

\[
N_{ci} = N_{1ci} + N_{2ci}, \quad (\text{Eq. 8})
\]

where \( N_{ci} \) is population of the civilization core for year \( i \); \( N_{1ci} \) is the ‘civilized’ population of Zone 1 for year \( i \); \( N_{2ci} \) is the ‘civilized’ population of Zone 2 for year \( i \).

At the same time:

\[
N_{1ci} = gF_1T_{ci}A_{1ci}, \quad (\text{Eq. 9})
\]

where \( A_{1ci} \) is the area of the part of Zone 1 controlled by civilization for year \( i \);

\[
N_{2ci} = gF_2T_{ci}A_{2ci}, \quad (\text{Eq. 10})
\]

where \( A_{1ci} \) is the area of the part of Zone 2 controlled by civilization for year \( i \).

Respectively,

\[
N_{bi} = N_{2bi} + N_{1bi}, \quad (\text{Eq. 11})
\]

where \( N_{bi} \) is population of the barbarian periphery for year \( i \); \( N_{2bi} \) is the ‘barbarian’ population of Zone 2 for year \( i \); \( N_{1bi} \) is the ‘barbarian’ population of Zone 1 for year \( i \).
Herewith,

$$N_{2bi} = gF_2T_{bi}A_{2bi},$$  \(\text{(Eq. 12)}\)

where \(A_{2bi}\) is the area of the part of Zone 2 controlled by ‘barbarians’ for year \(i\);

$$N_{1bi} = gF_1T_{bi}A_{2bi},$$  \(\text{(Eq. 13)}\)

where \(A_{1bi}\) is the area of the part of Zone 1 controlled by ‘barbarians’ for year \(i\).

The way of calculation of \(A_{1c}, A_{2c}, A_{2b}\) and \(A_{1b}\) variables employed by us in this model is described below (see Table 1 and Eqs 9, 10, 24, 25).

The total population of the World System for year \(i\) \((N_{wi})\) is calculated by means of the following equation:

$$N_{wi} = N_{ci} + N_{bi} + N_{3i}.$$  \(\text{(Eq. 14)}\)

Similarly to our general model of the World System development, mathematical description of technological dynamics is based upon the equation for technological growth proposed by Michael Kremer19 (Kremer 1993: 686):

$$\frac{dT}{dt} = hNT,$$  \(\text{(Eq. 15)}\)

where \(h\) is a constant (coefficient of technological innovative activity of population).

We assume that the diffusion of innovations proceeds from the civilization center of the World System to its barbarian periphery and from it to hinterland (1). Though in reality the diffusion of innovations from hinterland to periphery (2), from periphery – to center (3), as well as between various subzones of periphery (4) and hinterland (5) was also observed, after all the main flow of technological diffusion went in the first of the abovementioned directions (see, e.g., Chubarov 1991; Grinin A. and Grinin L. 2015, 2016) and we decided to refrain from the modeling of diffusion of technological innovations in other directions for the sake of simplicity of our model.

Thus, the following system of difference equations has been used in our model to model the technological development of the World System:

$$T_{ci} = T_{ci-1} + hN_{ci-1}T_{ci-1},$$  \(\text{(Eq. 16)}\)

where \(T_{ci}\) is the level of technological development of the civilization core of the World System for year \(i\);

$$T_{bi} = T_{bi-1} + hN'_{bi-1}T_{bi-1} + k(T_c - T_b),$$  \(\text{(Eq. 17)}\)

where \(T_{bi}\) is the level of technological development of the barbarian periphery for year \(i\); \(N'_{bi}\) – population of the barbarian Zone; \(k\) – a constant;

$$T_{3i} = T_{3i-1} + hN'_{3i-1}T_{3i-1} + l(T_{bi-1} - T_{3i-1}),$$  \(\text{(Eq. 18)}\)

where $T_{3i}$ is the level of technological development of hinterland (Zone 3) for year $i$; $N'_{3}$ is population of hinterland; $l$ is a constant.

As has already been mentioned above, we introduce an additional equation for description of dynamics of development of “barbarians’” military technologies to take into account the effect of the more rapid borrowing by “barbarians” of military technologies in comparison with peaceful technologies:

$$T_{mbi} = T_{mbi-1} + mN'_{bi-1}T_{mbi-1} + nH_{bi-1}(T_{ci-1} - T_{mbi-1}), \quad (Eq. 19)$$

where $m$ and $n$ are constants.

The effect that is of the most interest for us can be described by means of Eqs 17 and 19 by giving a higher value to $n$ coefficient in Eq. 19 in comparison with the value of $k$ coefficient in Eq. 17. This equation also describes an assumption that the rates of barbarians’ borrowings of military technologies grow along with the growth of their asabiyyahs.

We assume that all the urban population of the World System is concentrated in its civilizational core. For mathematical description of urbanization dynamics in the model the following equation is used:

$$u_{ci} = pT_{ci}, \quad (Eq. 20)$$

where $u_{ci}$ is the index of urbanization of the civilizational core (a share of urban population in the total population of civilization core), and $p$ is a constant. The possibility of approximation of $u_{ci} \sim pT$ follows from the equations of our compact model of the general World System development (Korotayev, Malkov, and Khaltourina 2007; Korotayev, Komarova, and Khaltourina 2007; Korotayev 2012, 2013). At the same time an empirical testing of this approximation was not carried out, and this testing was done by us. We use the World System Technological Development Index proposed by us earlier (Korotayev 2006a) for an empirical test of this hypothesis. Let us recollect that this index was calculated on the basis of Hellemans – Bunch database (Hellemans and Bunch 1988). In this database Hellemans and Bunch tried to record in chronological sequence all the main inventions and discoveries that had been made by the 1980s. As a value of the World System Technological Development Index for the moment $X$ we use the total number of inventions and discoveries which were made in the World System up to that moment.

The correlation between this World System Technological Development Index and the World System Urbanization Index calculated by us earlier (Korotayev, Malkov, and Khaltourina 2007: 122–127; Korotayev 2007; Grinin and Korotayev 2008: Ch. 4) looks as follows (see Fig. 4):
Thus, we find a rather strong and statistically significant correlation between these indices. The total number of urban population in the model is defined by the following equation:

\[ U = u_c N_c. \]  
(Eq. 21)

Finally, the World System Urbanization Index (a share of urban population in a total number of the World System population) \( u_w \) is defined by the following equation:

\[ u_w = \frac{U}{N_w}, \]  
(Eq. 22)

where \( N_w \) is the total population of the World System.\(^{20}\)

\(^{20}\) Thus, the World System urbanization appears here in our model as a purely dependent variable. Perhaps, it would make sense to consider its influence on some other key variable models (e.g.,...
Table 1 gives a summary description of the model:

**Table 1.** Compact mathematical model of influence of interaction of the civilizational center and barbarian periphery on the development of the World System (a detailed description)

<table>
<thead>
<tr>
<th>Variable symbol</th>
<th>Meaning</th>
<th>Equation number</th>
<th>Value for year i</th>
<th>Equation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>The territory of Zone 1 = Constant, in computer simulations the results of which are given below, has the value of 1 mln km$^2$</td>
<td>=</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>The territory of Zone 2 = Constant, 24 mln km$^2$</td>
<td>=</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$A_3$</td>
<td>The territory of Zone 3 = Constant, 96 mln km$^2$</td>
<td>=</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$F_1$</td>
<td>'Index of natural fertility' of Zone 1 = Constant, in computer simulations, the results of which are given below has the value 10</td>
<td>=</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$F_2$</td>
<td>'Index of natural fertility' of Zone 2 = Constant, 3</td>
<td>=</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$F_3$</td>
<td>'Index of natural fertility' of Zone 3 = Constant, 1</td>
<td>=</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$A_c$</td>
<td>The territory of 'civilization zone'</td>
<td>=</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_{c0} = A_1 = 1$ mln km$^2$. This variable cannot have negative values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_b$</td>
<td>The territory of the 'barbarian periphery'</td>
<td>=</td>
<td>(23)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_{b0} = A_2 = 24$ mln km$^2$. This variable cannot have negative values either</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{c1}$</td>
<td>The territory of the part of Zone 1, controlled by civilization</td>
<td>=</td>
<td>(9)</td>
<td></td>
</tr>
</tbody>
</table>
|                 | It is described by a version of Eq. 1; $A_{c0} = A_1 = 1$ mln km$^2$; it does not change while there is an expansion of civilization; if as a result of counterattack of barbarians they completely return Zone 2 to themselves, then $A_{c1} = A_{c1} + a(M_{c1} - M_{b1})$ till $A_{c1} (= A_c)$ reaches zero value (it is interpreted as a complete conquest of civilization by barbarians) or returns to value of 1 mln km$^2$ (it is interpreted on the rates of technological growth which was already made by M. Artzrouni and J. Komlos [Artzrouni and Komlos 1985] and that, in our opinion, might allow us to give a more exact description of technological dynamics of the World System in the basins of attraction of attractors $B_1$ and $B_2$), but in order to avoid excessive complication of the model we opt to refrain from this, though the action of this factor may be taken into account in future models.
<table>
<thead>
<tr>
<th>Variable symbol</th>
<th>Meaning</th>
<th>Equation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{2c}$</td>
<td>The territory of the part of Zone 2 controlled by civilization</td>
<td>$A_c - A_1$ if $A_c &gt; A_1$; 0 if $A_c \leq A_1$. (10)</td>
</tr>
<tr>
<td>$A_{2b}$</td>
<td>The territory of the part of Zone 2, controlled by ‘barbarians’</td>
<td>$A_b$ if $A_b \leq A_2$; $A_2$ (= in our case 24) if $A_b &gt; A_2$. (24)</td>
</tr>
<tr>
<td>$A_{1b}$</td>
<td>The territory of the part of Zone 1, controlled by ‘barbarians’</td>
<td>$0$ if $A_b \leq A_2$; $A_1 - A_c$ if $A_b &gt; A_2$. (25)</td>
</tr>
<tr>
<td>$M_c$</td>
<td>Military potential of civilization</td>
<td>$cN_cT_cH_c$ (2)</td>
</tr>
<tr>
<td>$M_b$</td>
<td>Military potential of the ‘barbarians’</td>
<td>$bN_bT_bH_b$. It is assumed that the value of the military participation ratio of ‘barbarians’ ($b$) is significantly higher than that for ‘civilized’ population. In computer simulations whose results are presented below, the value of $c$ is assumed to be equal to 0.05, and the value of coefficient $b$ is assumed to be equal to 0.2 (3)</td>
</tr>
<tr>
<td>$H_b$</td>
<td>Index of barbarians’ collective solidarity (asabiyah)</td>
<td>$H_{b_{i+1}} + e(A_{ci} - A_{ci-1}); H_{b_{i}} \geq 0$ (4)</td>
</tr>
<tr>
<td>$H_c$</td>
<td>Index of collective solidarity (asabiyah) of ‘civilized’ population</td>
<td>$H_{c_{i+1}} - e(A_{ci} - A_{ci-1}); H_{c_{i}} \geq 0$ (5)</td>
</tr>
<tr>
<td>$N_c$</td>
<td>Population of the civilizational core</td>
<td>$N_{1ci} + N_{2ci}$ (8)</td>
</tr>
<tr>
<td>$N_{1c}$</td>
<td>‘Civilized’ population of Zone 1</td>
<td>$gF_1T_cA_{1ci}$ (26)</td>
</tr>
<tr>
<td>$N_{2c}$</td>
<td>‘Civilized’ population of Zone 2</td>
<td>$gF_2T_cA_{2ci}$ (27)</td>
</tr>
<tr>
<td>$N_b$</td>
<td>Population of the barbarian periphery</td>
<td>$N_{2b} + N_{1b}$ (11)</td>
</tr>
<tr>
<td>$N_{2b}$</td>
<td>‘Barbarian’ population of Zone 2</td>
<td>$gF_2T_bA_{1Ni}$ (28)</td>
</tr>
</tbody>
</table>

---

21 It is obvious that the easiest way to model the dynamics of this variable is to give it the value of $A_i$ (that is 1 in our computer simulations) when $A_c \geq A_1$ and value $A_i$ when $A_c < A_1$. This method was also applied by us in real computer simulations for this and other similar variables ($A_{2c}, A_{2b} \text{ or } A_{1b}$).
<table>
<thead>
<tr>
<th>Variable symbol</th>
<th>Meaning</th>
<th>=</th>
<th>Value for year $i$</th>
<th>Equation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{1b}$</td>
<td>‘Barbarian’ population of Zone 2</td>
<td>$= gF_1 T_{bi} A_{bi}$</td>
<td>(29)</td>
<td></td>
</tr>
<tr>
<td>$N_3$</td>
<td>Population of Zone 3</td>
<td>$= gF_3 T_3 A_3$</td>
<td>(7)</td>
<td></td>
</tr>
<tr>
<td>$N_w$</td>
<td>Total population of the World System</td>
<td>$= N_c + N_{bi} + N_3$</td>
<td>(14)</td>
<td></td>
</tr>
<tr>
<td>$T_c$</td>
<td>Level of technological development of the World System civilizational core</td>
<td>$= T_{ci-1} + h N_{ci-1} T_{ci-1}$</td>
<td>(16)</td>
<td></td>
</tr>
<tr>
<td>$T_b$</td>
<td>The level of technological development of the barbarian periphery</td>
<td>$= T_{bi-1} + h N'<em>{bi-1} T</em>{bi-1} + k(T_c - T_b)$</td>
<td>(17)</td>
<td></td>
</tr>
<tr>
<td>$N'_{bi}$</td>
<td>Population in one subzone of ‘Barbarian Zone’ (with conditional area of each subzone being equal to 1 mln km$^2$)</td>
<td>$= N_{bi}/A_{bi}$ (note that the area of zones in our model is measured in mln km$^2$ therefore this division gives the population of ‘barbarians’ per 1 mln km$^2$)</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$T_3$</td>
<td>The level of technological development of the World System hinterland (= Zone 3)</td>
<td>$= T_{3i-1} + h N'<em>{3i-1} T</em>{3i-1} + l(T_{bi-1} - T_{3i-1})$</td>
<td>(18)</td>
<td></td>
</tr>
<tr>
<td>$N'_{3i}$</td>
<td>Population in one subzone of Zone 3 (with conditional area of each subzone being equal to 1 mln km$^2$)</td>
<td>$= N_{3i}/A_3$</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>$T_{mb}$</td>
<td>The level of development of military technologies of ‘barbarians’</td>
<td>$= T_{mbi-1} + m N'<em>{bi-1} T</em>{mbi-1} + n H_{bi-1}(T_{ci-1} - T_{mbi-1})$</td>
<td>(19)</td>
<td></td>
</tr>
<tr>
<td>$u_c$</td>
<td>Index of urbanization of the civilizational core (a share of urban population in the total population of the civilizational core)</td>
<td>$= p T_c, 0 \leq u_c \leq 0.9$</td>
<td>(9)</td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td>Total urban population</td>
<td>$= u_c N_c$</td>
<td>(21)</td>
<td></td>
</tr>
<tr>
<td>$u_w$</td>
<td>Index of the World System urbanization (a share of urban population in the total population of the World System)</td>
<td>$= U / N_w$</td>
<td>(22)</td>
<td></td>
</tr>
</tbody>
</table>
A typical dynamics generated by the model with average values of parameters and initial conditions is presented in Figs 5–7:

**Fig. 5.** Dynamics of the territory of the World System civilizational core generated by the main scenario of the model (mln km²)

**Fig. 6.** Dynamics of the World System urban population (millions) generated by the model, logarithmic scale
Fig. 7. Dynamics of the World System urbanization index (proportion of urban population in the total population of the World System) generated by the model, logarithmic scale

Note. Figs 5–7 show results of computer simulation with the following values of parameters and initial conditions: \( t_0 = 3000 \) BCE = –3000; \( A_1 = 1 \) mln km\(^2\); \( A_2 = 24 \) mln km\(^2\); \( A_3 = 96 \) mln km\(^2\); \( F_1 = 10 \); \( F_2 = 3 \); \( F_3 = 1 \); \( A_{1c0} = 1 \) mln km\(^2\); \( A_{2c0} = 0 \); \( A_{3c0} = 24 \) mln km\(^2\); \( A_{1b0} = 24 \) mln km\(^2\); \( A_{2b0} = 0 \); \( T_{c0} = 10 \); \( T_{b0} = 2 \); \( T_{a0} = 3 \); \( T_{1b} = 0.2 \); \( H_{c0} = 1 \); \( H_{b0} = 0.1 \); \( a = 0.012 \); \( b = 0.2 \); \( c = 0.05 \); \( e = 0.052 \); \( g = 0.05 \); \( h = m = 0.0000315 \); \( k = l = 0.000504 \); \( n = 0.00504 \); \( p = 0.00125 \).

Within this computer simulation one can distinguish the following phases:

Phase 1 (years 0–130 of the computer simulation). Vigorous accelerating expansion of civilization.

Accelerating expansion of civilization in this phase is generated by the following system of positive feedbacks (see Fig. 8):
Increase in population controlled by civilization ($N_c$) 

Growth of territory of civilization ($A_c$) 

Growth of military potential of civilization ($M_c$) 

Growth of technological potential of civilization ($T_c$) 

Reduction of territory of the barbarian periphery ($A_b$) 

Slowdown of the growth of ‘barbarians’ military potential ($M_b$) 

Reduction of population size of the barbarian periphery ($N_b$)

Fig. 8. System of positive feedbacks generating the accelerating territorial expansion of civilization during the first phase of the computer simulation

Thus, at this phase the growth of the civilization territory leads to the increase in its population, which results in increase in its military potential both directly (the size of the army [i.e. the number of soldiers] increases along with the increase in population size), and through acceleration of technological growth rates (allowing to supply the soldiers with more effective weapons); the increase in military potential of civilization leads to further increase in its territory which results in further acceleration of growth of its population, etc.; on the other hand, acceleration of the growth of the territory of civilization leads to a substantial reduction of the territory of barbarian periphery and consequently, decrease of population size and military potential of ‘barbarians’ that promotes further acceleration of growth of the territory of civilization, reduction of the territory of the barbarian periphery, etc.
At this phase one can observe the accelerated growth of population of the World System, the World System urbanization index and the urban population.

**Phase 2** (years 130–340 of the simulation). *Slowdown of expansion of civilization.*

The following system of negative feedbacks comes to the foreground during this phase: the growth of the civilization territory leads to the growth of asabiyyah of ‘barbarians’, which is expressed in increase in the level of their political culture and organization, and leads to the growth of their military potential both directly and through the acceleration of rates of borrowing of military technologies of civilization (including military and organizational and tactical innovations) which results in reduction of rates of growth of the civilization territory which, until it slows down to zero level, continues to lead (through the mechanisms mentioned above) to the growth of barbarians’ military potential and further slowdown of rates of territorial expansion of civilization (see Fig. 9).

Fig. 9. System of negative feedbacks generating slow-down of the territorial expansion of civilization at the second phase of computer simulation

---

22 First of all as a result of increasing diffusion of high technologies of civilization in the territories of Zone 2 subordinated by it, the growth of the carrying capacity there, and, therefore, the population.

23 In connection both with accelerating technological growth of civilization and with the growth of ‘civilized’ population percentage in the total World System population.

24 In connection both with the growth of urbanization in civilizational zone, and with the accelerated growth of its population as a result of the territorial expansion.

25 In historical reality we find the corresponding situation: as A. M. Khazanov notes, though nomads may seem barbarians for settled contemporaries, these ‘barbarians’ may be quite sophisticated in a political sense (Khazanov 2002: 54).
Nevertheless, at this phase the expansion of civilization proceeds at rather rapid (though more and more slowing down) rates; besides, a rather rapid (though slowing down) growth of population of the World System also continues. However, since year 154 of our computer simulation the absolute growth rates of population of the World System begin to decrease, but up to the end of Phase 2 they remain rather high. Since year 209 of the computer simulation the absolute growth rates of urban population also begin to decrease (remaining nevertheless rather high). During this phase, the growth rates of the World System urbanization index decrease almost twice (nevertheless remaining rather high if compared with the subsequent two phases).

**Phase 3** (years 340–510 of the simulation). *Expansion of civilization is exhausted and stops. Approximate power balance. The barbarian periphery begins its counterattack.*

During this phase, the territory of civilization in comparison to the territory of its barbarian periphery changes rather slowly, no more than 0.01 million km² per year (reaching at the inflection point, in year 408 of simulation, 48 km² per year). At the first stage of this phase the action of the above-mentioned mechanism of negative feedback leads to its logical conclusion – military potentials of civilization and barbarian periphery become equal to each other, and the rates of expansion of civilization reduce to zero level. However, the process of rather fast borrowing of military technologies of civilization by ‘barbarians’ continues. As a result military potential of the barbarian periphery begins to exceed that of civilization, and ‘barbarians’ start their counterattack. At the beginning it develops extremely slowly (83 km² during the first year); but the beginning of ‘barbarian counterattack’ leads to the formation of the system of positive feedbacks giving more and more noticeable results every year – acceleration of the growth of barbarian periphery territory leads to the acceleration of the growth of population of barbarian periphery, which in turn, leads to the increase in the military potential of ‘barbarians’ and even greater increase in the territory of barbarian periphery and consequently, to a greater increase in ‘barbarian’ population, *etc.*; on the other hand, acceleration of growth of the territory of barbarian periphery leads to a substantial reduction of the territory of civilization and consequently, to the decrease of civilization population, and military potential of civilization which promotes further acceleration of the growth of the territory of barbarian periphery, reduction of the territory of civilization, *etc.* (see Fig. 10).
Andrey V. Korotayev and Leonid E. Grinin

171

+\[ \text{Acceleration of the growth of population of the barbarian periphery (N_b)} \]

\[ + \]

\[ + \]

\[ \text{Acceleration of the growth of territory of the barbarian periphery (A_b)} \]

\[ + \]

\[ \text{Acceleration of the growth of military potential of ‘barbarians’ (M_b)} \]

\[ \text{Reduction of territory of civilization (A_c)} \]

\[ + \]

\[ + \]

\[ \text{Reduction of the rate of growth of military potential of civilization (M_c)} \]

\[ \text{Reduction of the population size of the civilizational center (N_c)} \]

Fig. 10. System of positive feedbacks generating the accelerating territorial counter-expansion of the barbarian periphery at the 4th and 5th phases of computer simulation

It is interesting to note that in our computer simulation by the time of the beginning of counterattack of barbarian periphery the total number of ‘barbarians’ (7.9 million) is almost three times less than the number of ‘the civilized population’ (23.4 million), and the index of their general technological development (3.7) is much lower than the level of technological development of civilization (12.1). At the same time the counterattack of barbarians appears to be possible due to a higher military participation ratio peculiar to them, and also due to the point that at the beginning of counterattack their asabiyyah is higher than the civilization asabiyyah, and their military technology is much higher (7.15) than the general level of their technological development (3.7).

The growth rates of the total population of the World System decrease in this phase almost three times from moderate 0.125 to 0.043 % per year. At the end of this phase there begins the slowdown in rates of technological growth of civilization. Urban population growth falls to 260 people per year, and growth of the index of urbanization – to 0.0002 % per year.


At this phase the mechanism of the positive feedback accelerating the counter-attack of the barbarian periphery works at full capacity. The territory controlled by the civilizational center is reduced by 2.5 million km². The population of
the civilizational center is reduced from 24.1 to 21.7 million people. This reduction is only partially compensated by the increase in population of the barbarian periphery and hinterland of the World System; as a result, the growth rate of total population of the World System falls from 0.043 to 0.01% per year. The rates of technological growth of civilization are reduced from 0.076 to 0.068% per year. Since year 614 of our computer simulation the continuing growth of urban population in the territory unoccupied by barbarians ceases compensating the reduction of the urban population as a result of counterattack of the barbarian periphery, and the total number of urban population begins to decrease. Even earlier (since year 577 of the simulation) the World System urbanization index begins to decrease.

**Phase 5** (years 680–935 of the simulation). *Slowdown of the expansion of the barbarian periphery.*

During this phase a system of feedbacks reducing the vigor of counterattack of barbarian periphery comes to the foreground.

**Fig. 11.** The system of negative feedbacks, generating the slowdown of territorial expansion of the barbarian periphery during Phase 5 of the computer simulation

Despite its slowdown, the continuing counterattack of the barbarian periphery throughout this phase is able to lead to very significant consequences. The area
of the territory controlled by the civilizational center is reduced almost twice. The population of the civilizational center falls from 21.73 to 15.97 million people. Until year 730 of the simulation this reduction is compensated in a lesser degree than earlier by the increase of population of the barbarian periphery and hinterland of the World System; therefore the growth rate in the World System total population reduces almost to zero. Later, the effect of the slowdown of expansion of the barbarian periphery begins to manifest itself, which against the background of continuing acceleration of growth rates of population of the barbarian periphery and hinterland leads to the renewal of the increase in growth rates of the total population of the World System (though it restarts growing very slow, showing the growth, say, only by 0.002 % between years 730 and 789 of our simulation, and even in 935 population growth rates of the World System remain extremely low, i.e., 0.027 % per year, while at the beginning of the first phase they were 0.37 % per year, i.e. they were ten times higher). Technological growth rates of civilization continue to decrease (from 0.068 to 0.05 % per year). The urban population declines from 403,000 to 344,000 people (at the same time the effect of the decline of the vigor of the barbarian counterattack begins to manifest itself – the absolute rates of decrease in urban population reach maximum in years 817–838, and then begin to decline). The World System urbanization index falls from 0.0116 to 0.0095 (though reduction rates of this indicator since year 865 of our simulation begin to decline too).

Phase 6 (years 935–2885 of the simulation). Expansion of the barbarian periphery reaches its peak. Approximate balance of forces. Civilization launches the counterattack.

During this phase, the territory of the barbarian periphery as compared to the territory of the civilizational core changes rather slowly, no more than 0.01 million km² per year (reaching at the inflection point just 0.5 km² per year in year 2047 of our simulation). At the first stage of this phase (years 935–2047), the action of the negative feedback described in Fig. 10 produces its logical conclusion: military potentials of civilization and barbarian periphery become equal to each other, and expansion rates of the barbarian periphery decline to zero. However, the rates of technological development (including the growth rates of military technologies) of civilization continue to outpace those for the barbarian periphery. As a result, the military potential of civilization begins to exceed that of the barbarian periphery, and the civilization begins its counterattack (with the level of 1.23 million km²) in year 2048 of our simulation. At first it proceeds very slowly (only 1.5 km² during the first year of the ‘counterattack’); but the beginning of civilization counterattack leads to the formation of a system of positive feedbacks (described in Fig. 8), giving more and more noticeable results every year, – growth of the civilization territory
leads to the increase in its population which leads to the growth of military potential both directly and through acceleration of technological growth rates; the growth of military potential of civilization leads to further increase in its population, which leads to further acceleration of growth of its territory, etc. As a result, the territory of civilization grows from the level of 1.23 million km² in year 2048 to 2.94 million km² by the end of this phase at increasing (but still, in general, rather slow) rates.

The growth rates of the total population of the World System continue to increase throughout all the 6th Phase, but by very slow rates, increasing from 0.027 to 0.12 % per year, still remaining lower than the rates characteristic of the beginning of Phase 1. The population of the civilizational center continues to fall until year 1438 of our simulation, declining from 15.97 to 12.73 million, and then its growth resumes, and, by the end of the phase, the civilization population reaches 38.5 million, considerably exceeding the level reached at Phases 1–3. The reduction of technological growth rate of civilization continues till year 1439 of the simulation (declining from 0.05 to 0.04 % a year), and then this rate begins to grow rapidly, reaching the level of 0.12 % per year by 2885 (i.e., it considerably exceeds the level reached at Phases 1–3). The World System urban population continues to decline till year 1169 of the simulation (decreasing from 344,000 to 320,000 people, then its growth is resumed, gradually accelerating, and, by the end of Phase 6, the urban population of the World System reaches 2,340,000). The World System urbanization index continues to fall much longer – till year 1600 of our simulation, declining from 0.0095 to 0.0074; and then it begins to grow with gradual acceleration, reaching the level of 0.0206 by the end of Phase 6.

Phase 7 (years 2885–3209 of the simulation). As a result of a vigorous counterattack civilization completely subordinates Zone 2, absorbing the whole barbarian periphery.

During this phase, there is a rapid growth of all the modeled indicators of the level of development of the World System. By year 3065 of the computer simulation the urban population of the World System reaches the level of 10 million, and the World System urbanization index exceeds 10 % in 3123. In reality at this level, civilization already have to contact with the extensive hinterland of the World System (which during our simulation managed to achieve rather high levels of population and technological development), whereas the World System hinterland would transform into the new barbarian periphery of civilization, which, with certain values of parameters, could lead to a new counterattack of the barbarian periphery at a higher level. However, it would lead to ad-
ditional complication of the model from which we have decided to refrain at this stage.\textsuperscript{26}

It is interesting that correlation between technological development and urbanization of the World System generated by this model is surprisingly similar to what we have seen above (see Fig. 4) for the empirical estimates of the level of technological development of the World System, on the one hand, and the level of its urbanization, on the other (see Figs 12–14):

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{correlation.png}
\caption{Correlation between values of the World System technological development index ($T$) and the World System urbanization index generated by the model ($u$)}
\end{figure}

\textsuperscript{26} This model also does not describe the withdrawal of the World-System from the blow-up regime. In theory, in our case it might be possible, having described basic population dynamics by means of the following equations: $\frac{dN}{dt} = r \frac{dT}{dt} \cdot (1-l)$, $\frac{dN}{dt} \leq 0.04$, $\frac{dl}{dt} = s \frac{dT}{dt} \cdot (1-l)$ (where $l$ is a proportion of literate population, and $r$ and $s$ are constants), and basic urbanization dynamics – by means of the following equations: $\frac{du}{dt} = v \frac{dT}{dt} \times (u_{\text{lim}} - u)$ (where $u_{\text{lim}}$ is a maximum possible share of urban population, and $v$ is a constant); justification of the equations of this type, see, e.g., in the following works: Korotayev 2006a; Korotayev, Malkov, and Khaltourina 2007; Korotayev, Komarova, and Khaltourina 2007. However, we decided not to do it in order to avoid the excessive complication of the model especially since it does not include the description of withdrawal of the World-System from the blow-up regime, and mathematical models with such a description have been already offered and published by us earlier (see e.g., Korotayev 2006; Korotayev, Malkov, and Khaltourina 2007; Korotayev, Komarova, and Khaltourina 2007).
Fig. 13. Correlation between the empirical estimates of the index of the World System technological development ($T$) and the empirical estimates of the World System urbanization index ($u$) (3500 BCE – 1970 CE)

In this figure, most part of the curve up to 430 BCE looks like a solid black spot. However, after ‘zooming’ in this spot, it is possible to see that the curve in this sector has a form that is surprisingly similar to the one of the whole graph – a sort of fractal effect (see Fig. 14):
As we see, the shapes of all the three curves are amazingly similar: in the initial part of the figure even rather small technological growth is followed by very noticeable growth of urbanization of the World System. Then it is followed by a pronounced interval when further technological growth is accompanied by decrease of the urbanization level, which is changed by an interval where technological growth is accompanied by slow growth of urbanization which is followed by a stretch where technological growth is accompanied by rapid growth of urbanization with a subsequent new interval of relative slowdown.

It is worthy of note that formal indicators of correlation for model values of these two variables ($R = 0.95, R^2 = 0.903$) are almost identical to those for correlation calculated by us (see note to Fig. 4) for empirical values of these variables.
Perhaps, it is not a pure coincidence, as this correlation both in our model and in reality in many respects was generated by similar mechanisms. Thus, the impact of the barbarian periphery on the central (‘civilized’) territories of the World System in the 1st millennium CE led to noticeable barbarization and de-urbanization of many of these zones as a result of which the World System urbanization index considerably decreased. At the same time, the rates of technological growth (both in our model and in reality) decreased, but technological growth did not stop completely since at that time new inventions and discoveries continued to be made (especially in those zones of the World System which underwent de-urbanization least of all). As a result, at that time the decrease in the level of urbanization was followed by some (albeit decelerated) growth of the level of its technological development, which caused an apparent negative correlation reducing general level of the general positive correlation between the variables in question.

Note that, in our computer simulation, the final phases of conquest of the barbarian periphery by civilization are not deprived of some dramatism. The matter is that, not long before the end of the full conquest of the barbarian periphery by civilization, its asabiyyah falls to zero (‘dizziness with success’?) which leads to a fast and powerful counterattack of the barbarian periphery, burst of civilization asabiyyah and even more rapid final submission of the barbarian periphery (Fig. 15):

![Fig. 15. Dynamics of the civilization territory before the final conquest of barbarian periphery (years 3170–3209 of our computer simulation)](image)

In our opinion, quite a probable scenario of events is modeled here; however, fortunately, it does not appear to have been ever realized in real history of the World System (though the events of 9/11, perhaps, can serve here as a slightly resembling analogue).
The numerical study of influence of parameter values on the dynamics of our model shows that the key parameters determining the length of phases are as follows: the coefficient of innovative activity \((h)\) defining the rates of technological growth; the coefficient of territorial expansion \((a)\) in Eq. 1; the coefficient of borrowing technologies of civilization by ‘barbarians’ \((k)\) in Eq. 17 and in particular military technologies \((n)\) in Eq. 19; the coefficient of dynamics of asabiyyah \((e)\) in Eqs 4 and 5, and also the relationship between the coefficient of military participation of barbarians \((b)\) and civilization \((c)\).

Small reduction of the value of coefficient \(h\) with respect to the value mentioned in the note to Figs 6–8 leads to some reduction of duration of phases 1–3 and significant increase in duration of other phases, but first of all of Phase 6 (‘relative equilibrium of forces’); with reduction in coefficient of technological development \((h)\) the rates of historical development are slowing down (see Fig. 16).

![Graphs showing the influence of coefficient \(h\) on the territory](image)

**Fig. 16.** Dynamics of the territory of the World System civilizational center generated by model (millions km\(^2\)) with a small decrease in the coefficient of technological growth \((h)\)

Small reduction of the coefficient of territorial expansion \((a)\), or small increase in values of the coefficient of asabiyyah dynamics \((e)\), coefficient of borrowing of civilization technologies by ‘barbarians’ \((k)\) and, in particular, military technologies \((n)\), and also small increase in gap between the coefficients of military participation of population of the barbarian periphery and civilizational core \((b/c)\), small reduction of initial values of the level of technological development of civilization \((T_{c0})\), its territories and asabiyyah, and also small increase in the initial values of the level of technological development, territory, and asabiyyah of the barbarian periphery produce similar results.
Fig. 17. Dynamics of the territory of the World System civilizational center generated by model (millions km$^2$) with a small decrease in the coefficient of the territorial expansion ($a$)

$$a) \ a = 0.0120 \quad b) \ a = 0.0094$$

Fig. 18. Dynamics of the territory of the World System civilizational center generated by the model (in millions km$^2$) with a small increase in the coefficient of asabiyyah dynamics ($e$)

$$a) \ e = 0.0520 \quad b) \ e = 0.0528$$

Fig. 19. Dynamics of the territory of the World System civilizational center generated by the model (in millions km$^2$) with a small increase in the value of overall coefficient of the borrowing of civilization technologies by 'barbarians' ($k$)

$$a) \ k = 0.000504 \quad b) \ k = 0.000517$$
Fig. 20. Dynamics of the territory of the World System civilizational center of generated by the model (in millions km²) with a small increase in the value of the coefficient of borrowing of civilization military technologies by ‘barbarians’ (n)

Fig. 21. Dynamics of the territory of the World System civilizational center generated by the model (in millions km²) with a small increase in the gap between the military participation ratio of barbarian periphery and civilizational core (b/c)

Fig. 22. Dynamics of the territory of the World System civilizational center generated by the model (in millions km²) with a small reduction of initial value of the level of civilization technological development (T₀)
The above described small changes of parameters and initial conditions of the model lead to the reduction of sizes of territory to which control of civilization extends during the first wave of its expansion (Phases 1–2.5), and also to increase in duration and intensity of counterattack of the barbarian periphery (Phases 2.5–5.5). As a result, the zone remaining under control of civilization at the maximum of barbarian counter-expansion is reduced. The further change of parameters and initial conditions in this direction leads to significant changes of the overall picture of dynamics and implementation of a significantly different scenario. If (as a result of the barbarian expansion) the civilization zone is reduced to the level below 1 million km², it means that in the respective model simulation the barbarian periphery manages to conquer a part of the nuclear civilization zone (Zone 1) with an especially high natural productivity. As one can see in the model simulation, this leads to a very pronounced strengthening of barbarians (even at the phase when their counterattack approaches its exhaustion) and to a very serious weakening of civilization. In our simulations, the civilization could only launch a counterattack if barbarians managed to take no more than 1–2 % of Zone 1. Otherwise sharp strengthening of barbarians together with sharp weakening of civilization leads to a new acceleration of barbarian expansion and rapid final conquest of civilization by barbarians. Thus, a new phase is added (Phase 6’ – the phase of the new acceleration of barbarian expansion and final conquest of the civilization by barbarians), and Phases 6.5–7 disappear.

Further reduction of the coefficient of territorial expansion (a) reduces the size of the territory which is under control of civilization at the peak of its territorial expansion, but at the same time postpones ‘barbarian occupation’, prolonging the life of civilization (see Fig. 23).

![Fig. 23. Dynamics of the territory of the World System civilizational center generated by the model (in million km²) at considerable decrease in the coefficient of territorial expansion (a)](image-url)
On the other hand, further reduction of the coefficient of innovative activity \((h)\) or increase in the values of the coefficient of asabiyyah dynamics \((e)\), the coefficient of borrowing technologies of civilization by ‘barbarians’ \((k)\) (and, in particular, military technologies \((n)\)), as well as further increases in the gap between coefficients of military participation of population of the barbarian periphery and civilizational core \((b/c)\), further reduction of the initial values of the level of technological development of civilization \((T_c)\), its territory and asabiyyah, and also further increase in initial values of the level of technological development, territory, and asabiyyah of the barbarian periphery lead to reduction of the ‘life of civilization’, to its more rapid conquest by barbarians (Figs 24–30):

![Graphs showing the impact of different coefficients on territory size.](image)

**Fig. 24.** Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a considerable decrease in the coefficient of technological growth \((h)\)

**Fig. 25.** Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a significant increase in the coefficient of asabiyyah dynamics \((e)\)
Fig. 26. Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a significant increase in the value of general coefficient of borrowing of civilization technologies ($k$) by ‘barbarians’

Fig. 27. Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a significant increase in the value of coefficient of borrowing of military technologies of civilization by ‘barbarians’ ($n$)

Fig. 28. Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a significant increase in the gap between military participation ratios of the population of barbarian periphery and civilizational core ($b/c$)
It is obvious that with such values of parameters the model describes quite a real scenario. Indeed, with a certain set of parameters the expansion of civilization could create such a powerful barbarian periphery that its counterattack could be able to destroy this civilization (the classic example here is the conquest by barbarians of Rome, expansion of which in many respects gave barbarians that very strength which eventually helped them to break down their formidable opponent [see, e.g., Turchin 2005]).

Beyond a certain limit we get a scenario of more and more rapid conquest of civilization by barbarians already without a phase of the initial civilizational expansion (see Fig. 30).\(^\text{27}\)

\(^{27}\) We should also note that the change of parameters of the model in this direction is after all meaningful only to a certain degree. Say, the coefficient of military participation of barbarians \((b)\), cannot by definition exceed 1.0, but even 1.0 describes an unrealistic scenario because the entire population of any society (including, as we know, newborn babies and very old men and women) cannot actually take part in combat. Similar restrictions also exist for all other parameters and initial conditions of the model.
In this case we have already a rather banal scenario of the final conquest of civilization by barbarians whose military superiority has already been determined by initial parameters and initial conditions, while civilization (with the same parameters and initial conditions) has no chance to effectively resist the barbarian expansion.\footnote{At the same time, say, if the conquest of civilization by ‘barbarians’ within the model is brought about by assigning to ‘barbarians’ a higher initial value of asabiyya ($H$), it can already be interpreted within Ibn Khaldun’s tradition (see, e.g., Ibn Khaldun 1958, 2004; Batısheva 1965; Ignatenko 1980; Alekseev and Khaltourina 2004; Turchin 2003, 2007; Korotayev and Khaltourina 2006; Korotayev 2006c, 2007c, 2007d; Inan 1933; Mahdi 1937) as the conquest of a low-asabiyyah civilization by high-asabiyyah barbarians, which does not ‘end the history’, but begins its new round, a new ‘Khaldunian’ dynastic cycle, but this will be quite a different model.}

In case of little change of parameters and initial conditions in the opposite direction\footnote{That is with a small increase of the coefficient of technological growth ($b$) and the coefficient of territorial expansion ($a$), or with a small reduction of the values of the coefficient of asabiyyah dynamics ($e$), the coefficient of borrowing of civilization technologies by ‘barbarians’ ($k$) and, in particular, military technologies ($n$), and also with a small decrease of the gap between military participation ratios of the barbarian periphery and civilizational core ($b/c$), with a small increase in the initial values of the level of technological development of civilization ($T_{te}$, Fig. 22), its territory and asabiyyah, and also with a small reduction of the initial values of the level of technological development, territory and asabiyyah of the barbarian periphery.} (in relation to the parameters and initial conditions described in the note to Figs 5–7) we will see in our computer simulations some increase in du-
ration of phases of the initial expansion of civilization, the territory occupied by it during expansion (as well as the territory retained by it at the peak of ‘barbarian’ counterattack), and reduction of duration of all subsequent phases, and in particular Phase 6 (‘relative balance of forces’) (Figs 31–37):

Fig. 31. Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a small increase in coefficient of technological growth ($h$)

Fig. 32. Dynamics of the area of the territory of the World System civilization center generated by the model (in millions km²) with a small increase in the coefficient of territorial expansion ($a$)
Fig. 33. Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a small reduction of the coefficient of asabiyyah dynamics (e).

Fig. 34. Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a small reduction of the value of the general coefficient of borrowing of civilization technologies by ‘barbarians’ (k).
Fig. 35. Dynamics of the territory of the World System civilization center generated by the model (in millions km$^2$) with a small reduction of the value of the coefficient of the borrowing of military technologies of civilization by ‘barbarians’ ($n$)

Fig. 36. Dynamics of the territory of the World System civilization center generated by the model (in millions km$^2$) with a small reduction of gap between military participation ratios of barbarian periphery and civilizational core ($b/c$)

Fig. 37. Dynamics of the territory of the World System civilization center generated by the model (in millions km$^2$) with a small increase in the initial value of the level of technological development of civilization ($T_{c0}$)
In case of further changes of parameters in this direction (and, in particular, with their combined changing) one can observe a further increase in duration of phases of the initial expansion of civilization, the area of the territory occupied by it during the first expansion phases (as well as the territory kept by it at the peak of ‘barbarian’ counterattack), and reduction of duration of all subsequent phases, and in particular Phase 6 (‘relative balance of forces’). Gradually duration of phases 3–6 reduces to zero, and they disappear, whereas phases 1–2 eventually merge with phase 7. As a result, we have only phase 1, and further change of parameters in this direction leads only to reduction of the period of time which is required for civilization to conquer its barbarian periphery (Figs 38–41):

Fig. 38. Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a significant increase in the coefficient of technological growth ($h$)
Fig. 39. Dynamics of the territory $e$ of the World System civilization center generated by the model (in millions km²) with a considerable reduction of the coefficient of asabiyyah dynamics ($e$).

- a) $e = 0.0475$
- b) $e = 0.0450$
- c) $e = 0.0400$
- d) $e = 0.0100$

Fig. 40. Dynamics of the territory of the World System civilization center generated by the model (in millions km²) with a large increase in the initial value of the level of technological development of civilization ($T_{c0}$).

- a) $T_{c0} = 12$
- b) $T_{c0} = 15$
A Model of Center-Periphery Interaction

Fig. 41. Dynamics of the territory of the World System civilization center generated by the model (in millions km$^2$) with a simultaneous considerable reduction of the coefficient of asabiyah dynamics ($e$) and a large increase in the initial value of the level of technological development of civilization ($T_{c0}$).

Let us note that in all cases we deal with quite real scenarios of interaction between civilizational center and barbarian periphery. In fact if a technologically highly developed (and, in addition, rapidly developing) civilization came in contact with such a periphery which was extremely undeveloped technologically and incapable of borrowing technologies of civilization rather quickly, it led to the accelerated conquest by civilization of its periphery that was incapable to produce any effective resistance to advancing civilization (the British colonization of Australia presents a rather salient example of such a scenario).

Thus, depending on the given parameters and initial conditions the offered model can describe mathematically seven significantly different scenarios of the interaction between civilizational center and barbarian periphery (see Table 2):
Table 2. Scenarios of interaction between civilization center and barbarian periphery described by the model depending on the given parameters and initial conditions

<table>
<thead>
<tr>
<th>Description of scenario</th>
<th>Values of parameters and initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Accelerated conquest of the barbarian periphery by civilization</td>
<td>Very high values of coefficients of technological growth (h), territorial expansion (a), very low values of coefficients of dynamics of asabiyyah (e), borrowing of civilization technologies by ‘barbarians’ (k) and, in particular, military technologies (n), very small gap between coefficients of military participation of population of the barbarian periphery and civilizational core (b/c), very high initial values of the level of technological development of civilization (Tc0), its territory (Ac0) and asabiyyah (Hc0); very low initial values of the level of technological development (Tb0), territory (Ab0) and asabiyyah (Hb0) of the barbarian periphery</td>
</tr>
<tr>
<td>2. The phase of the accelerated expansion of civilization is followed by the phase of its slowdown caused by more and more effective counteraction of ‘barbarians’ who, nevertheless, are not able to mount a counterattack</td>
<td></td>
</tr>
<tr>
<td>3. The phase of accelerated and deep expansion of civilization and phase of its slow-down is followed by rather short-term and shallow counterattack of ‘barbarians’ which is succeeded by the phase of final submission of barbarian periphery</td>
<td></td>
</tr>
<tr>
<td>4. The phase of still accelerating but less deep expansion of civilization and the phase of its slowdown is followed by the phase of a long and deep offensive of ‘barbarians’ which is succeeded by a long phase of relative balance of forces which is followed by the phase of final submission of the barbarian periphery by civilization</td>
<td></td>
</tr>
<tr>
<td>5. The phase of less and less accelerating and deep expansion of civilization and the phase of its slow-down is followed by the phase of the long and deep offensive of ‘barbarians’ which is gradually slowed down by more and more effective counteraction of civilization which, nevertheless, is not able to prevent its complete conquest by barbarians</td>
<td></td>
</tr>
</tbody>
</table>
6. The phase of rather shallow slowing down expansion of civilization is followed by the phase of accelerated advance of ‘barbarians’ which results in the conquest of civilization by the ‘barbarians’

7. As a result of a rapid, increasingly accelerating offensive, the barbarians conquer civilization

<table>
<thead>
<tr>
<th>Description of scenario</th>
<th>Values of parameters and initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Very low values of the coefficient of technological growth ( h ) and the coefficient of the territorial expansion ( a ); very high values of the coefficient of dynamics of asabiyyah ( e ), the coefficient of borrowing of civilization technologies ( k ) by ‘barbarians’ and, in particular, military technologies ( n ); a wide gap between military participation ratios of the barbarian periphery and civilizational core ( b/c ); very low initial values of the level of technological development of civilization ( T_{c0} ), its territory ( A_{c0} ) and asabiyyah ( H_{c0} ); very high initial values of the level of technological development ( T_{b0} ), territory ( A_{b0} ) and asabiyyah ( H_{b0} ) of the barbarian periphery</td>
</tr>
</tbody>
</table>

Thus, the scenario displayed above in Figs 5–7 and most precisely describing the influence of interaction between the civilization center and barbarian periphery on the World System development is intermediate among the above-described scenarios. One can suppose that this is not a coincidence. In fact, there are some grounds to maintain that this interaction could have the historically attested impact only with intermediate values of parameters. Barbarians could not confront effectively the expansion of civilizational and launch a massive counterattack with very low values of coefficients \( e, k, n \) and \( b \), i.e., if their collective solidarity were not sufficiently amplified under the influence of pressure of civilization, if they had not been able to adopt vitally important technologies of civilization (including military ones) quickly enough, if they had not have a much higher military participation ratio. On the other hand, a too high value of these parameters would not have made the expansion of civilization of the World System possible, or would have even led (at especially high values of these parameters) to rapid conquest of civilization by barbarians before the civilization expansion could start. One can say the same about the values of all the other significant parameters of the model, and also about the values of the initial conditions (e.g., if at the time of clash of civilization with the barbarian periphery the relative level of its technological development had been too low, it would not have been able to begin expansion at all, and with an extremely high value it would have rapidly subordinated the periphery that would be incapable of offering any effective resistance).
On the other hand, we can easily see that practically all the scenarios described above were observed in the history of the World System at the level of some specific civilization zones and their peripheries. In many respects this is precisely why the total dynamics of the World System appears to be the closest to the intermediate scenario of the model.

Of course, in our case the fit of theoretical curve with empirical estimates is still far from being ideal – which is not surprising since nobody would claim that the interaction between civilization core of the World System and its barbarian periphery was the only factor that defined the characteristic form of the curve of world urbanization dynamics.

What is really surprising for us, is the fact that the offered model after all appeared capable to describe the general form of this dynamics so precisely (though, of course, imperfectly). Hence, this suggests that the interaction between the civilization core of the World System and its barbarian periphery was really an important factor making a notable contribution (until very recently) to giving the peculiar form to the curve of the world urbanization dynamics.

The proposed model suggests that in the history of the World System development the important component of World System phase transitions $A_1$ and $A_2$ could be not only the movement of its core to a new level of complexity, but also the formation of barbarian periphery of an essentially new type capable to offer much more effective resistance to civilization expansion and to mount successful counterattacks that apparently could make a noticeable contribution to the formation of the ‘attractor effect’ during periods $B_1$ and $B_2$.

There are also some more points that seem to be capable to explain why this model which takes into account only one factor of the World System dynamics (and not always the most important one), was able to generate the curve so well describing empirically observed historical macrodynamics. The fact is that this model, most likely, describes general development logic of the World System within which the hyperbolic growth of its civilizational core creates powerful forces restricting this growth at certain phases of the World System development (or, to be more exact, moving the World System to a new, lower, hyperbolic trajectory) (see, e.g., Korotayev 2006d, 2007; Korotayev, Malkov, and Khaltourina 2007; Grinin and Korotayev 2009b).

For example, one of such factors (which influenced dynamics of the World System in Phase $B_2$ probably, not lesser, if even not greater than the effect of interaction between civilization center and barbarian periphery) was earlier described by us as follows:

The growth of the World System population by the end of the 1st millennium BCE up to 9-digit numbers produced a breeding ground that led to an almost inevitable appearance of a new generation of more lethal and epidemically destructive pathogens that could not reproduce themselves in smaller populations (Diamond 1999: 202–205; McNeill 1993; Koro-
It is very important that the logic of action of this factor is extremely similar to the logic of action of the factor analyzed in this article. In the both cases, the hyperbolic growth of civilization creates powerful forces that block this growth. On the other hand, the pressure of the barbarian periphery was able to stimulate the growth of military potential of civilization, whereas pathogenic attacks on the World System eventually stimulated development of health care technologies which allowed the World System to repulse these attacks more successfully and renew its hyperbolic growth.30

The logic of this factor is similar to the logic of another factor (which, apparently, had an even greater influence on the World System dynamics World System in phase $V_1$, than the effect of interaction between the civilizational center and barbarian periphery) – the factor of environmental degradation under the influence of the hyperbolic growth of civilization. This factor is most evident in the history of ancient Mesopotamia (whose curve of urbanization dynamics in the 4th – early 2nd millennia BCE defined the general shape of the World System urbanization dynamics curve to a very considerable extent). As is well known, the explosive growth of civilization in this region led to catastrophic soil salination in its most developed zone, in Lower Mesopotamia, which, in turn, led to decrease in rates of demographic and urbanization growth here up to negative values in the middle of the 3rd millennium. On the other hand, environmental degradation stimulated the technological growth here which in the 1st millennium CE led to the restoration of the carrying capacity values here up to the levels of the beginning of the 3rd millennium BCE, and then to its noticeable expansion (Dyakonov 1983: 272, 330; Chubarov 1991; Roberts 1998: 175).

30 It appears appropriate to recollect at this point that this growth continued up to the early 1970s when the World-System started to withdraw from the hyperbolic growth regime (i.e., the blow-up regime) due to force created by its hyperbolic growth, but these were forces of another kind (Korotayev, Malkov, and Khaltourina 2006a, 2006b, 2007).
These facts explain to some extent a paradox that can be noticed above in Figs 1–2, 5–7, and 12–14. The point is that in order that the interaction of civilization center and barbarian periphery could produce an effect that is similar to the one that is actually observed, the territory under the control of civilization should be decreased manifold and remain at an extremely low level for a very long period of time, which apparently did not happen in reality. Apparently, what was not ‘consumed’ by ‘barbarians’ at ‘sinks’ in Figs. 1–2 corresponding to the phases $B_1$ и $B_2$ was mostly ‘eaten away’ additionally by pathogenic attacks and environmental degradation.

Finally, let us dwell upon some other factors that seem to be relevant for modeling of the long-term dynamics of the World System. One of these factors was described by us earlier as follows:

Some hint here seems to be suggested by mathematical models (0.11)-(0.12) and (0.13)-(0.14)31 described in the Introduction. According to these models any long-term decrease of per capita surplus ($S$) must lead to the decrease of population growth rates and, hence, the slow-down of technological growth. In the meantime, by the end of the Axial Age we seem to observe a World System trend towards the decline of precisely this indicator. This was connected not with decline of production, but rather with the growth of $m$, the per capita product that is necessary for the population reproduction with zero growth rate, the ‘minimum necessary product’ (MNP). In the 1st millennium BCE the rapid population growth sustained the hyperbolic growth of the complexity of sociopolitical infrastructures (on the other hand, of course, the hyperbolic population growth was also sustained by a hyperbolic growth of sociopolitical complexity – once more we are dealing here with the positive feedback phenomenon). However, the radical increase in sociopolitical complexity meant a radical increase in the MNP, as the substantial expenses necessary for the normal functioning of these sociopolitical infrastructures should be regarded, in this context, as a part of the minimum necessary product (rather than as surplus). Indeed, by the end of the 1st millennium BCE the World System population reached 9-digit numbers; even a simple reproduction (at zero growth rate) of so huge a population required maintenance of normal functioning of all those infrastructures (transportation, judicial, administrative and other such subsystems). Within such a context, if the product produced this year by a peasant is only sufficient to secure the survival of himself and his household, but not sufficient to pay any taxes, it is impossible to say that this peasant has produced this year the minimum necessary product. In fact, what he has produced this year is smaller than the MNP. Indeed, as the experience of post-Axial centuries showed on numerous occasions, in supercomplex agrarian societies the decrease of per capita

---

31 These numbers refer to equations presented in the Introduction (Korotayev, Malkov, and Khalto-urina 2006b).
production (usually as a result of relative overpopulation) down to a level that did not allow the population to pay taxes led to the disintegration of sociopolitical infrastructures and demographic collapse (see Korotayev, Malkov, and Khaltourina 2006h: Chapters 1–432). There are grounds to maintain that the rapid growth of the MNP in the 1st millennium BCE exceeded the growth of the equilibrium per capita production, which resulted in the long-term decrease of real $S$, and, hence, the decrease of the World System population growth rates. On the other hand, it led to the decrease of sociopolitical system stability, and, hence, to the increase in the importance of the role of cyclical and chaotic components of the macrohistorical dynamics in comparison with the trend component (ibid.: 159–160; 2007: 206–207).

Obviously, in this case we also deal with a force that was created by the hyperbolic growth of the World System and that blocked for some time the further hyperbolic growth of the World System.

In conclusion, we would like to make a few additional comments. We are well aware of the fact that many assumptions of our model simplify (sometimes even over-simplify) the situation observed in historical reality. It is worth pointing out some of such assumptions and points which should be developed, systematized and considered in future generations of models of interaction between civilization center and barbarian periphery.

First of all, one should note that one of the strongest simplifications of the model was that the World System consisted of only one civilization and only one barbarian periphery (though in history we naturally deal with a number of civilizations and barbarian peripheries surrounding them). Taking into consideration the multiplicity of civilizations may become one of the leading directions of further development of our model. It could also be possible to present some typology of both civilizations, and barbarian peripheries and to use this typology for explanation of characteristic features of the World System dynamics both within basins of attraction, and during phase transitions.

For example, it would be worth dividing barbarian periphery into two types. The first one is represented by barbarians-agriculturalists; the second one is by barbarians-herders (nomads). This point is important with respect to the level of the world urbanization, because nomadic population constituted a rather small fraction of the global population, whereas barbarians-farmers could constitute a substantially higher percentage (e.g., according to some estimates, the population of Gaul before the Roman conquest was between 5 and 10 million [see, e.g., Braudel 1995: 61–62]). Note that the transition of nomads, semi-nomads and extensive farmers to settled intensive agriculture appears to have greatly influenced the population dynamics of the World System. Apparently, period $B_1$ is

characteristic of such transition (in particular, in India among Indo-Aryans, in Iran, among the Dorians in Greece, in a number of other areas of Europe, etc.). In other words, transformations of barbarian periphery could significantly change proportions in the world population and its urbanization due to population growth of the barbarian periphery (which is considered only partially in the present version of the model).

At the initial stage (the first two thirds of the 1st millennium CE), period B₁ was connected with absorption by the civilization center of the huge number of barbarians who intruded in its territory. However, it appears necessary to take here into account the difference between complex agrarian societies of B₁ period and supercomplex agrarian systems of B₂ period in respect of the economic role of cities.

In typology of civilization centers it appears important to distinguish between irrigation and non-irrigation civilizations. It is very important due to significant differences in the processes of their urbanization. In particular, three functions of cities in the regions of river civilizations were more developed than in other places: economic, redistributive and sacral. And that fact made the cities actually a part of agrarian technology within civilization whereas cities could not act as such an integral element elsewhere. Moreover, the politically centralizing role of the cities in irrigation civilizations was also higher, and non-irrigation agricultural civilizations had more opportunities to remain decentralized without great losses for efficiency of functioning of their economy. As a result, the elite of irrigation civilizations was to a greater extent urban, and the elite of non-irrigation civilizations might be rural to a greater extent (e.g., in the medieval Islamic Middle East the elite was generally urban, and in medieval Europe it was rural to a greater degree). Thus, the character of civilizations and barbarian periphery at different stages might influence the level of urbanization.

Distinguishing types of civilizations and barbarian peripheries could help us to achieve a significant clarification of typical reasons and variants of military rivalry between civilizations and barbarians. In particular, it is possible to preliminarily outline the reasons which forced civilizations and states to make the expansion to the barbarian periphery (the reasons of attacks of barbarians against civilizations will be discussed later):

1) Attempts to eliminate dangerous centers of constant concern and attacks. It was one of the main reasons of Chinese campaigns against Xiongnu, Russian campaigns against Cumans, etc.

2) Attempts to bring back territories occupied by barbarians.

3) Campaigns to seize slaves and booty (were characteristic of a number of African states). This type of campaigns includes the attacks for the purpose of getting tribute and necessary goods, strategic raw materials, etc. For example, the expansion of Russians to the North and Siberia was largely determined by their need for furs.
4) Campaigns with hegemonic purposes – ‘in pursuit of power’. It was an important reason for which the Persian king Darius I tried to conquer Scythians.

5) Campaigns and conquests with some strategic aim (improvement of the situation, acquirement of favorable and convenient means of communication, elimination of potential danger, etc.). Among such cases one can mention conquests of Gaul and Dacia by Romans which were triggered by a special political situation, the involvement of these people into complex political game, attempts of their threat to Romans or their allies, etc. In particular, a cause for Caesar's campaign to Gaul, as we know, was that the Sequani, which suffered defeat from the Aedui, called Germans (Suevi and others) for help in campaign led by Ariovist who not only successfully defeated the Aedui, but also began to enslave the Sequani. These events served as an important pretext for Romans to interfere in affairs of the Gauls (see: Caesar. The Gallic War I: 31–39). Conquests which were made under such circumstances were also very characteristic of Europeans of the Modern Period.

6) Seizure of agricultural lands. One of examples are Charlemagne's campaigns against the German and other barbarians, another one is the expansion of the German knights to the Baltic States (a motive to seize lands for agricultural purposes was especially salient in this case). One can add the expansion of Carthage to Corsica, Sardinia and to Spain. It appears appropriate to notice in all these cases we deal with the expansion to the territories occupied by agriculturalists. And therefore it is very important to take into account (as this is done in our model) that the territory of the barbarian periphery was divided, at least, into two types:

- the territory more economically attractive to the civilization;
- the territory less economically attractive.

It is obvious that expansion to the second type of territories was not important for civilization in itself (this only became important when this periphery disturbed it). Not without reason China could refrain from such expansion for a long time.

On the other hand, the systematic transition to intensive non-irrigation agriculture (that was observed just in the 1st millennium BCE) might fully strengthen such expansion in its different types (for example, such was the case with Greek colonization which can be considered as one of the types of non-military or partly military expansion). In general, if the expansion of civilization to the barbarian periphery was in any way successful and prolonged, then as a rule it led to assimilation of barbarians. One should note that in future models it would be worthwhile introducing mathematical description of processes of barbarians’ assimilation in a more explicit way.

However, civilization could not assimilate those barbarians who lived in marginal (unsuitable or almost unsuitable for agriculture) zones.
Probably, expansion of civilization to the periphery was not vital for a number of civilizations and played a subordinate role for them. However, such expansion was very important for the World System as a whole, which is shown in our model.

One should also define different types of expansion of civilization to the barbarian periphery. For example, it is possible to speak about assimilating expansion which might happen without essential resistance of the peripheral peoples. On the other hand, the more lands the civilization was bringing into its economic turnover, the more often it faced a more resistant and intractable (and at the same time less and less attractive economically) barbarian periphery.

Probably, in future models it would make sense to include mathematical description of the expansion of some barbarians to the territory of others. If the lands of nomads are often economically unattractive for civilizations, they are almost always economically attractive to other nomads. But nomads can also attack barbarians-farmers (as it happened with the Huns who attacked the Goths). And such kind of expansion quite often causes great changes by the ‘domino’ principle leading to general expansion of barbarians against the civilization center.

We have already mentioned above that the issue whether barbarians need or need not centralization for their successful expansion against the civilization has no unambiguous solution.

Successful wars and especially conquests of neighboring states by barbarians were quite often connected with successful centralization of barbarians (at least temporary) around some leader. It is relevant for the Xiongnu, the Mongols, the German tribes, the Huns, and many others. Sometimes as a result of these perturbations the center strengthened and a large chiefdom emerged. However, if this centripetal movement was insufficiently steady to become permanent, the life cycle of a new large polity was short. Such fragile formations as the Slavic Samo’s state’ (Lozny 1995: 86–87), the Germanic tribal unions of Maroboduus (among the Markomanni), Ariovistus (among the Suevi), Arminius (among the Cherusci), Claudius Civilis (among the Batavians) (Neusykhin 1968: 601–602; Oosten 1996); the Hunnish ‘empire’ of Attila (Korsunsky and Günther 1984: 105–116); the Getae and Dacians union led by the ‘king’ Burebista (Fedorov and Polevoy 1984) and others usually disintegrated after the death of the charismatic leader (and sometimes during his life as it happened to Maroboduus). Sometimes one could observe the decline of the supreme power in analogues even before the death of such a leader, especially if there was a strong and self-willed elite.

While considering the interaction between barbarians and civilization one should take into account the ambiguity of the solution of the issue of military and technological superiority of the latter over the former. Anyway, such a superiority was not always sufficient as barbarians could compensate it by sud-
denness or other advantages. Before the wide diffusion of iron, civilizations having expensive and highly technological weapons (e.g., bronze arms) could not always successfully resist barbarians either, especially if civilizations were internally weakened.

It is also worth mentioning the features of barbarian political formations in comparison with the states of civilization zone.

Of particular interest are barbarian analogues of the early state where the political organism did not have rigidly fixed territory, more precisely, where socio-political organism can change its territory rather easily. Of course, it is much less characteristic for the states where the state control over a certain territory is almost obligatory. The states seldom change the core of their territory.34 No matter how the borders of states change, the core usually remains the same, whereas barbarians, for example the Hungarians or the Goths (Shchukin 2005, etc.), were able to move thousands of kilometers ‘in search for their home’. In some cases, an important cause for such migrations was constituted by population pressure. ‘A sharp increase in population density in settled agricultural societies is well known, compared to the epoch of hunter-gatherers. It increased almost a hundredfold’ (Masson 1976: 102–104, 189; 1980: 182–183). The importance of demographic growth in increasing role of wars in the relations between societies is great. And wars can also lead to the development of new political forms (e.g., Grinin and Korotayev 2012; Turchin 2015). Not without a reason Robert Carneiro constantly emphasizes that increased population pressure can lead to wars and conquests, therefore state organization emerges in certain cases and under certain circumstances (Carneiro 1970, 1978, 2000a, 2002; 2006, 2012; see also: Lewis 1981). High population pressure quite often caused migrations and wars even among more populous state analogues. One of the most known examples is the huge polity of the Visigoths which suffered from relative overpopulation already under Hermanaricus in the middle of the 4th century CE (Shchukin 2005: 219), which was the most important reason for their migration to Byzantium (note that this episode is often considered as the beginning of the Great Migration Period).

One should also take into consideration the point that barbarian periphery being more poor and backward aspires to the robbery of civilization to a greater extent, than vice versa. Arnold J. Toynbee (1991) (somewhat simplifying the reality) said that ‘external proletariat’, i.e. barbarian peoples, posed threat for civilization having ‘nothing to lose’ just as Marx’ proletariat, but they could gain lots of assets by attacking the civilization. This, of course, strengthened the barbarians’ pressure.

34 One of such extremely rare examples was represented by the South African Dutch who moved their states far away from the British in 1836–1839 (see, e.g., Büttner 1981: 189–190).
It also appears appropriate to mention the role of wars in the life of barbarian societies. As some researchers note, wars among hunter-gatherers were observed a bit less frequently than among barbarians (Lesser 1968: 94; Korotayev, Malkov, and Khaltourina 2006b; Korotayev, Komarova, and Khaltourina 2007: 143, 148). There is also a certain significant correlation between extreme living conditions and a low level of aggression (Kazankov 2002). Therefore, nomadic hunter-gatherers who live under especially extreme conditions can be mostly characterized as relatively peaceful societies. It greatly differs from the behavior of many nomadic herders living under extreme conditions and having small population density. The latter are just distinguished by especially high levels of aggression. Thus, aggression considerably increases with the transition to barbarism. As Karl Marx and Friedrich Engels (who generally underestimate the role of wars in history) note in the *German Ideology* (2004 [1845–1846]: 89), ‘with the conquering barbarian people war itself is still… a regular form of intercourse, which is the more eagerly exploited as the increase in population together with the traditional and, for it, the only possible, crude mode of production gives rise to the need for new means of production’.

It should also be noted that for barbarians (unlike for states/civilizations) external exploitation can play a significantly more important role than internal exploitation (see, e.g., Kradin 1992; Grinin 1997, 2003a). Many researchers emphasize that very often exploitation begins not inside, but outside the society since the stranger is protected by neither tradition, nor custom. External exploitation strengthens inequality and, undoubtedly, promotes the development of polityogenesis. For example, K. Pietkiewicz notes that before the formation of the Lithuanian state two basic strata were found among the Lithuanians: free farmers and warriors (nobles) who were called *kunigai*, i.e., ‘dukes’, ‘lords’. ‘Dukes’ attained material well-being and high positions in society through predatory wars, to a lesser extent through gathering tribute from their own population (Pietkiewicz 2006: 306).

Acts of violence often played a very important role in the life of culturally simple farmers and herders. Such actions were one of the most important ways for individuals to raise their social status. For example, N. A. Butinov writes the following about the Papuans:

> There were two ways of advance up to high rankers: peaceful and military; the second, probably, prevailed. The aspirant for a higher status brought together a group of men. Under his leadership they attacked neighboring villages, plundered, killed, subordinated survived to their power. The reason for attack was easy to invent (e.g., black magic, theft of pigs, abduction of women, disputes on the territory etc.). Justification of murder of aliens was not necessary, as it was considered a good deed. Intercommunal wars took place very often (Butinov 1995: 62).
Wars played a significant role even in the formation of simple chiefdoms (see Carneiro 1970, 1981, 2004, 2012). On the whole the ideas to occupy neighboring settlements by force of arms; to take prisoners and to force them to work as slaves; to demand periodic payment of a tribute’ were rather wide spread (though not universal) at that time (see, e.g., Carneiro 2004; see, however, Zinkina et al. 2016).

Emergence or diffusion of some important technological innovations could lead to great changes in politogenesis and could become its accelerator. As a result, intensive politogenetic processes could begin in those places where politogenesis was strongly delayed or impossible at all before. As has been mentioned above, the role of such an accelerator might have been played by the diffusion of iron metallurgy, progress in using of mount or draught animals. Emergence of the cavalry, iron weapons, etc. also promoted ‘intensification’ of military operations, strengthened the role of wars in politogenesis.35

The role of wars in some respects is especially significant in the development and transformation of pre-state and state analogue barbarian societies. For most of relevant barbarian peoples, wars became the most important factor of their transformation into the state. A striking example was the state of the Zulu which at the beginning of the 19th century transformed from a conglomerate of chiefdoms into an empire very quickly (just in two or three decades) (see, e.g., Davidson 1968: 5; 1984: 161; L'vova 1984: 47; Maquet 1974: 91; Potekhin 1954: 545; Gluckman 1987 [1940]: 29). Wars also played a prominent role in generating many important innovations which could be the source of ‘energy’ promoting the most powerful expansions and important reasons for fast military victories changing the World System map. In particular, there are some opinions that at the beginning of the second millennium CE evolution of nomadic military science reached such a level of development that essentially affected military art of other societies and civilizations of Eurasia (Khudyakov 1991: 3). The idea that at the beginning of the 13th century Mongols won in many respects due to some important innovations introduced in their military organization has become generally accepted (see, e.g., Khrapachevsky 2005).

One should also note here that there are significant divergences concerning the role of wars in the processes of state formation. In the discussions over the state formation, as James Ambrosino notes, ‘the role and influence of external social factors, i.e. such impetuses which are created by contacts with foreign societies, were practically ignored’ (Ambrosino 1995: 54). Among modern researchers the theory of influence of wars on politogenesis and state formation was most systematically and consistently developed by Robert Carneiro

35 E.g., with the diffusion of horses in the Great Plains, theft of horses became the main reason of wars among the Amerindian peoples (such as the Omaha) in the North American prairies. Mobility of riding horses led to the point that the Utah, Apaches, Navaho, Shoshone and others began to raid peoples they never faced before the penetration of horses (Dennen 1995: 429).
Andrey V. Korotayev and Leonid E. Grinin

(1970, 1978, 1981, 1987, 2000a, 2000b, 2000c, 2002, 2003, 2004, 2006, 2012; see also Godiner 1991; Turchin 2010, 2015; Grinin and Korotayev 2012). However, these ideas were not generally accepted. So, though the role of trade, cultural and other borrowings and influence in political anthropology is considered more adequately to some extent, the significance of wars for processes of politogenesis in general and formation of the state in particular is still underestimated by many researchers (if not by most of them).

It should be noted that in the late 19th century and the early 20th century most scholars estimated the role of wars in the state formation much higher than it was done later. For example, P. F. Preobrazhensky considered war as the inevitable concomitant of the government (2005: 154). K. Kautsky (1931) contrary to the Marxist doctrine had to eventually recognize that conquest was the most important cause for state formation. L. Gumplowicz and F. Oppenheimer were the most famous researchers who believed that the state formed due to a simple conquest of one nation by another. It is also worth mentioning Gustav Ratzenhofer.36 Subsequently such approaches were rejected as too primitive not without some reason. Nevertheless, mistakes of a century ago do not mean that wars did not play an important role in politogenesis. Just on the contrary. At least, we do not know any case when the military factor was absent (at least in some form) during emergence and formation of the early state. By the military factor we mean a situation, connected both to waging wars in this or that way (aggressive or defensive), and with preparation for them, or with a direct conquest (submission) of some communities by means of military force (for more details see Grinin 2007a; Grinin and Korotayev 2012). At the same time the emergence of many states (even in terms of creation of truly new political and administrative forms) often took place on the basis of military structures, customs, institutes, for example military camps of young men, troops, personal guard, security structures, etc. (see, e.g., L'vova 1995: 161; Orlova and L'vova 1978; Miller 1984: 191; Bocharov 1991: 70).

Probably it would make sense to include in the future models a mathematical description of the point that the diffusion of civilization happened not only due to military or settler expansion of civilizations, but also due to development of certain zones of barbarian periphery that quite often passed civilizational threshold directly in the course of offensive on the neighboring civilization or during defense against this offensive. So, A2 period is characterized by transformation of the extensive part of barbarian agricultural periphery into civilization which was an important factor of the additional growth of urban population.

If we take into consideration these factors, as well as factors of cyclic and stochastic dynamics, this may allow us to develop such mathematical models that may be capable of giving more exact description of the long-term dynamics of the World System.

References


гала. Возрастные группы народов Восточной Африки / К. П. Калиновская, с. 133–143. М.: Наука).


Korotayev A. V. 2006b. Social History of Yemen of the 10th Century BCE – 20th century CE. Moscow: KomKniga/URSS.


Andrey V. Korotayev and Leonid E. Grinin

Законы истории. Вековые циклы и тысячелетние тренды. Демография. Экономика. Войны. М.: КомКнига/URSS).


