4
Political Violence and Evolutionary Game Theory: A Methodological Introduction

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Abstract

Classical game theory is one of the basic methods of scientific analysis of political phenomena. The models developed on this basis are used in the studies of electoral and legislative behaviour, in the analysis of processes of forming political coalitions, and in the analysis of issues related to democratization, national security and armed conflicts. Evolutionary game theory has developed from classical game theory. This theory is referred to in this article, which presents selected possibilities of using single-population evolutionary models in studies of political violence transmission. On the basis of the analysis of two population variants, the article describes the changes in the prevalence of selected behavioural traits and answers the questions regarding the asymptotic states of evolutionary processes and their stability. The study uses Hawk-Dove and Hawk-Dove-Retaliator type games. The calculations were carried out using the R program.

Keywords: *political violence, evolutionary game theory, mathematical models, Hawk-Dove game, Hawk-Dove-Retaliator game.*

Where there is conflict over values, access to resources or power, violent behaviour usually occurs. For this reason, violence and politics must be seen as inextricably connected social phenomena. The main objective of political violence is to influence the political process. This goal is achieved through actions that can cause physical and/or mental harm to political opponents. Such activities include insulting, intimidating, blackmailing, blocking communication routes, occupying government buildings, kidnapping, physical assault, political murder, terrorism, and in extreme cases also participation in a coup d'état, revolution or civil war (Mider 2013: 706–707; 2017: 43–44).

Political violence is of interest to the representatives of multiple scientific disciplines, such as political science (Kalyvas 2006; Tausch 2007; Hatemi and McDermott 2012; McDermott *et al*. 2013), sociology (Malešević 2010; Vertigans 2011), psychology (Ofreneo and de Vela 2006; Michaels 2017), law

(Schabas 2000; Walker 2011) or history (Bercé 1987; Raaflaub 2007). Research is conducted within the framework of theoretical and methodological orientations, which are often extremely different. Some researchers limit their analysis to the cultural paradigm only, trying to find the determinants of political violence among the values and norms adopted in a given society (*e.g.*, Ofreneo and de Vela 2006), while others recognize the relevance of extra-cultural factors, for example neurobiological or genetic ones (see *e.g.*, Hatemi and McDermott 2012; McDermott *et al*. 2013). This article focuses not on the causes of political violence or its possible consequences, but on the process of spreading violent behaviour in human populations. It shows how evolutionary game theory models may be used in the studies of the transmission of political violence.[1]

The article consists of three parts. The first part provides a concise introduction to evolutionary game theory. It describes such notions as the Nash equilibrium, evolutionarily stable strategies and replicator dynamics. In the second part, selected models are analyzed. The starting point for the analyses was the Hawk-Dove interaction scheme (the game is also known as Chicken). The article is concluded with a summary.

A Crash Course in Evolutionary Game Theory

A research in classical game theory involves interactive situations, *i.e.* situations involving two or more individuals. Depending on the research objectives and adopted assumptions, they can be people, companies, political parties, states, *etc.* These individuals are called players, and the situation they find themselves in – a game. Classical game theory assumes the rationality of the players, which means that they should choose strategies that produce the best possible results while being aware that their opponents are doing exactly the same. Conversely, according to its original purpose, evolutionary game theory allows for modelling interactions between genetically determined organisms within specific biological populations. Individuals from the same population usually participate in a game where the same resources are at stake. Two adult males may compete for a female, herd domination or fertile territories with abundant resources. The aim of the fight is, therefore, to take control of the resources, which significantly increase the ability to survive and to pass on one's genes to the next generations. The result of such a game is determined by the strategies (genetically determined behavioural traits) applied by individual players. As a result, the strategies with higher average payoffs (fitness) gain an advantage in subsequent generations due to the increased reproduction rate of

[1] Evolutionary game theory is used to investigate a wide range of social and political phenomena that are based on the use of violence. Let us just mention fundamentalism (Arce and Sandler 2003), religious and ethnic conflicts (Qin *et al*. 2014; Luo, Chakraborty, and Sycara 2011) or civil violence (Goh *et al*. 2006; Quek, Tan, and Abbass 2009).

the players using these strategies (Maynard Smith and Price 1973: 15; Vincent and Brown 2005: 72–75).

The central concept of classical game theory is the Nash equilibrium (Nash 1950, 1951). It describes a situation in which none of the players gains anything by changing their strategy if the other players remain constant in their choices. In a formal structure, the profile of strategy s^* is a Nash equilibrium if:

$$u_i\left(s_i^*, s_{-i}^*\right) \geq u_i\left(s_i^{'}, s_{-i}^*\right) \quad \forall s_i^{'} \in S_i, \forall i. \qquad \text{(Eq. 1)}$$

An analogous concept for evolutionary game theory is evolutionarily stable strategy (ESS) (Maynard Smith and Price 1973; Maynard Smith 1982). If ESS is used by the entire population, each small group of mutants (individuals born with new phenotypic traits) or migrants (individuals coming from the outside) using a different strategy will be eliminated from the population. According to the formal definition, strategy s^* is evolutionarily stable (assuming $\forall s' \neq s^*$) when:

$$u\left(s^*, s^*\right) > u\left(s', s^*\right), \qquad \text{(Eq. 2)}$$

or

$$u\left(s^*, s^*\right) = u\left(s', s^*\right) \Rightarrow u\left(s^*, s'\right) > u\left(s', s'\right). \qquad \text{(Eq. 3)}$$

The first condition means that strategy s^* must be the best response to itself. This is correct for a strict Nash equilibrium. Therefore, if a pair of strategies (s^*, s^*) creates a strict Nash equilibrium, then s^* is an evolutionarily stable strategy. From the second condition, we learn that if s' is another best response to s^*, then s^* must be a better response to s' than s' is to itself.

The ESS concept is applicable when players come from the same population and play a symmetric game. Bimatrix game symmetry means that both sides of the conflict have the same number of pure strategies to choose from and that the second player's payoff matrix is a transposed payoff matrix of the first player. In such a game, the Nash equilibrium is symmetric if both players select the same strategies. Formally, a bimatrix game $G = (A, B)$ with dimensions $m \times n$ is symmetric if $m = n$ and $b_{ij} = a_{ji}$ for all $i, j = 1, \dots, m$. Therefore, the Nash equilibrium (p^*, q^*) of G is symmetric if $p^* = q^*$.

The Nash equilibrium is static. The concept describing the process of achieving equilibria in the population is replicator dynamics.[2] It allows us to follow the changes in the relative frequency of occurrence of particular strategies. Replicator dynamics is calculated using the following formula:

$$\dot{x}(t) = \frac{dx(t)}{dt} = x(t)\left[U_A(t) - \bar{U}(t)\right], \qquad \text{(Eq. 4)}$$

where $x(t)$ means the percentage of the population playing strategy A in time t,

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[2] Replicators are relatively durable, self-replicating structures that determine strategic behaviours in a game. In biological games, genetic replicators are subject to selection and mutation, while in non-biological games this applies to social or cultural replicators.

$U_A(t)$ means the fitness provided by the selection of strategy A in time t, and $\bar{U}(t)$ is the average fitness of the entire population in time t.

The following relationships can be observed between the fitness provided by the choice of strategy A and the average fitness of the entire population:

(i) $U_A(t) = \bar{U}(t)$ – the population is in a static state, therefore $\frac{dx}{dt} = 0$;

(ii) $U_A(t) > \bar{U}(t)$ – the proportion of individuals choosing strategy A increases in the population, therefore $\frac{dx}{dt} > 0$;

(iii) $U_A(t) < \bar{U}(t)$ – the proportion of individuals choosing strategy A decreases in the population, therefore $\frac{dx}{dt} < 0$.

The concept of replicator dynamics was developed for biological applications. It can also be used, however, in social science studies. We then assume that the basis of evolutionary changes is not genetic variability, but imitation of behaviours of higher utility.[3] In this perspective, replicator dynamics is a basic example of imitative dynamics (Sandholm 2010a: 30–31; 2010b: 153–154).

In the next section, the models showing exemplary applications of evolutionary game theory in studies of political violence are analyzed.[4]

Models and Analyses

Let us assume that the subject of analysis is the interactions among the population consisting of the representatives of the political elite of state X.[5] This population is so numerous and relatively homogeneous that the evolutionary processes that occur within it are of a statistical nature. The population is non-spatial, so at any time any player has the same likelihood of entering a conflict of interests with any other player. Game participants have to choose one of two strategies: Hawk (H) – extremely non-cooperative attitude, escalating conflict (*e.g.*, blackmail, actions discrediting the opponent in the eyes of public opinion, physical violence *etc.*) or Dove (D) – striving to reach an agreement by way of concessions. The decision regarding the selection of H or D strategy is made

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[3] One of the direct consequences of the differences in the course of the evolutionary process is the need for a different interpretation of payoffs. In biological games, they are considered in terms of fitness, *i.e.* the number of offspring inheriting the strategy of their parents. Conversely, in social games, we do not talk about fitness, but about utility, which is related to the choice of a given strategy.

[4] It should be remembered that they are far-reaching simplifications. Therefore, they do not show how the phenomenon under investigation actually takes place nor do they explain why it does; they only show how it could occur under certain arbitrary assumptions.

[5] Interaction schemes used in the article are taken from *The Logic of Animal Conflict* (Maynard Smith and Price 1973) and *Evolution and the Theory of Games* (Maynard Smith 1982). The simulations were carried out using the Evolutionary Games library (Gebele and Staudacher 2017) available within the R program (R Core Team 2017).

based on the observations regarding which of the two strategies results in better payoffs (has higher utility).

In evolutionary games, payoffs are considered in terms of the benefits (*b*) and costs (*c*) resulting from a specific action. In the analyzed game, benefits can be identified with resources such as positions or money (in perspective – being able to influence effectively political processes), while costs are damaged reputation, loss of credibility and public trust (in perspective – losing one's position and influence, and in extreme cases, criminal responsibility).[6] Let us assume in our model that fighting may lead to more loss than gain ($b < c$). Parameters *b* and *c* were therefore assumed on levels 3 and 5. Below, there is a general model of the Hawk-Dove game and the model under analysis.

$$\begin{array}{ccc} & H & D \\ H & \frac{b-c}{2} & b \\ D & 0 & \frac{b}{2} \end{array} \qquad \begin{array}{ccc} & H & D \\ H & -1 & 3 \\ D & 0 & 1\frac{1}{2} \end{array}$$

The row player and the column player choose *H* or *D*. The following combinations of strategies are possible in the game:

1) Hawk-Hawk – the entering of Hawks into the interaction means a ruthless fight generating high costs on both sides;

2) Hawk-Dove (Dove-Hawk) – a conflict between a politician playing *H* and a politician choosing *D* will end with the withdrawal of the latter without taking up a fight; as a consequence, the Hawk will grab the entire reward pool, while the Dove will leave without losing or gaining anything;

3) Dove-Dove – a competition between Doves will result in the peaceful division of resources into two equal parts.

There is no doubt, therefore, that the most favourable situation will be for a Hawk to meet a Dove. Two Doves will come out of the interaction equally satisfied. The worst-case scenario, however, will be for a Hawk to compete with another Hawk.

The game has two asymmetric Nash equilibria in pure strategies (*H*, *D*) and (*D*, *H*). In addition, the game payoff structure generates one symmetric equilibrium in mixed strategies, which corresponds to mixed ESS. Let us assume that the row player selects strategy *H* with probability *p*, and strategy *D* with probability $(1 - p)$. Assuming the indifference of the column player regarding the payoffs resulting from the selection of *H* or *D*, we look for a *p* which fulfils the following:

$$-1p+3(1-p)=0p+1\frac{1}{2}(1-p), \qquad \text{(Eq. 5)}$$

[6] If we assume that country X has democratic rules for the election of its political representatives, the benefits and costs can be directly reduced to the level of public support measured in percentage points.

which results in $p=\frac{3}{5}$. Because the payoffs of the game are symmetric, the mixed equilibrium requires that each player decides on *H* with a frequency of $\frac{3}{5}$, and on *D* with a frequency of $\frac{2}{5}$. In such a case, the expected values of payoffs will be equal and will amount to $\frac{3}{5}$ units of utility each. Therefore, if both players choose *H* and *D* with the probabilities provided by the mixed equilibrium, the composition of the population will stabilise temporarily. This condition will persist until a new mutation (political behaviour) occurs.

Fig. 1 presents the replicator dynamics of the Hawk-Dove game. The share of the Hawk strategy in the population is presented on the x-axis, on the y-axis – $\frac{dx}{dt}$.

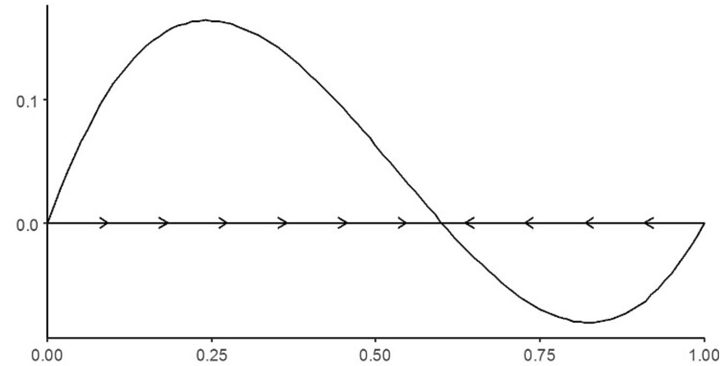

Fig. 1. Phase diagram of replicator dynamics in a Hawk-Dove game

Source: author's calculations.

Formally, the replicator dynamics of this game can be expressed with the use of the following equation:

$$\dot{x}(t)=\frac{dx(t)}{dt}=x(t)\left[-4x(t)+3-\left(1\frac{1}{2}-2\frac{1}{2}x(t)^{2}\right)\right], \qquad \text{(Eq. 6)}$$

which, when simplified, results in

$$\dot{x}=\frac{dx}{dt}=x(1-x)\left(1\frac{1}{2}-2\frac{1}{2}x\right). \qquad \text{(Eq. 7)}$$

Equation (6) shows that the changes in the population are proportional to the difference between the utility of Hawks over time *t* and the average utility of the entire population over time *t*. In the phase diagram (see Fig. 1), there are three stationary (critical) points of replicator dynamics: $x = 0$, $x = \frac{3}{5}$ and $x = 1$, in which the population takes a static form. At point $x = 0$, the population is homogeneous; it consists only of individuals choosing the Dove strategy. The utility of Doves in this state is equal to the average utility of the entire population. This condition, however, is not stable, as even a slight disturbance (mutation) will lead to an increase in the number of individuals playing extremely uncooperatively. The growth barrier is determined by point $x = \frac{3}{5}$. A similar process will take place in the opposite direction. The appearance of non-violent individuals in a population composed exclusively of Hawks (point $x = 1$) will result in a decrease in the number of the latter, with a simultaneous increase of the former. Therefore, one can see that the only stable point of replicator dynamics is $x = \frac{3}{5}$.

The basic Hawk-Dove model can be further specified by taking into account other behaviours. Let us now imagine a population whose representatives have three strategies to choose from: Hawk (*H*), Dove (*D*) or Retaliator (*R*). The politicians who select *R* adjust their behaviour to the behaviour of their opponents. When a Retaliator meets an individual playing *H*, it will try to escalate the conflict, which will generate equally high costs for both sides. In contact with the Dove, the Retaliator will behave like a Dove. It will receive, however, a slightly higher payoff than its opponent. Conversely, the rivalry between two Retaliators will end in an equal division of the disputed goods.

As in the previous model, players interact randomly to receive payoffs and then compare them with the payoffs of other players. Strategies bringing higher average payoffs are subject to adaptation by subsequent players. Below, there is a general model of the Hawk-Dove-Retaliator game and the model under analysis. As was the case in the Hawk-Dove game, parameters *b* and *c* are assumed at levels 3 and 5.

	H	*D*	*R*
H	$\frac{b-c}{2}$	b	$\frac{b-c}{2}$
D	0	$\frac{b}{2}$	$\frac{2b}{5}$
R	$\frac{b-c}{2}$	$\frac{3b}{5}$	$\frac{b}{2}$

	H	*D*	*R*
H	-1	3	-1
D	0	$1\frac{1}{2}$	$1\frac{1}{5}$
R	-1	$1\frac{4}{5}$	$1\frac{1}{2}$

The game has three symmetric Nash equilibria. In addition to the equilibrium in mixed strategies (*I, I*) in which $I = \frac{3}{5}H + \frac{2}{5}D$, it has an equilibrium in pure strategies (*R, R*) and another equilibrium in mixed strategies (*Q, Q*) in which $Q = \frac{3}{13}H + \frac{250}{481}D + \frac{120}{481}R$. It appears that, depending on the starting point (initial frequency of particular strategies), the population will evolve either to the state of $\frac{3}{5}$ Hawks and $\frac{2}{5}$ Doves, as was the case in the Hawk-Dove game, or to the homogeneous state – 100 % Retaliators. In other words, there are two ESS in the model: $I = \frac{3}{5}H + \frac{2}{5}D$ and R. The evolution of the population is presented in the de Finetti diagrams in Fig. 2.

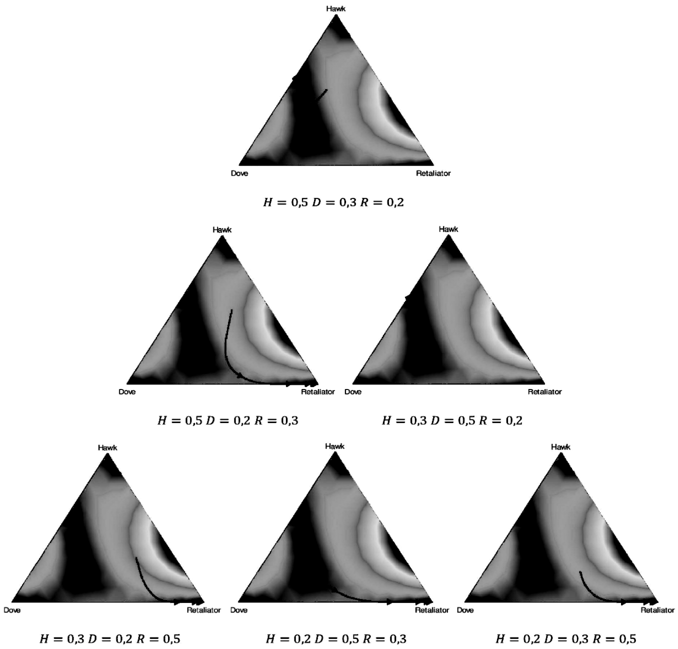

Fig. 2. Examples of trajectories of population evolution in the Hawk-Dove-Retaliator game depending on the initial proportions of the strategies. The models must meet the following assumptions: $H + D + R = 1$ and $H \geq 0$, $D \geq 0$, $R \geq 0$. Red indicates where the replicator dynamics accelerates and blue indicates where it slows down.

Source: author's calculations.

At the initial state of the population, the prevalence of one strategy was 50 %, while the occurrence of the other strategies was 30 % and 20 %. The simulations show that in four cases, strategy *R* eliminated strategies *H* and *D*. In each of them, the initial share of *R* was 30 % or more. In two variants, however, strategy *R* lost to its competitors, which resulted in the determination of an evolutionarily stable state, which is a stable proportion of the number of politicians playing *H* and choosing *D* in the population.

Summary and Discussion

The article presents selected possibilities of using single-population evolutionary models in studies of political violence. The analyses began with the basic Hawk-Dove model, assuming that the costs of conflict were higher than the expected profit ($b < c$). In this configuration, an evolutionarily stable state was established, which was a stable proportion of Hawks and Doves in the population. If we were to assume, however, that the expected profit from victory outweighed the losses from defeat ($b > c$), the Hawk strategy would eliminate the Dove strategy from the population entirely. In other words, the Hawk strategy would be an evolutionarily stable strategy. In the next step, the basic model was made more specific, taking into account the strategy of Retaliators. As a result, the population evolved either to a homogeneous state – 100 % Retaliators – or to a state in which 60 % of the population were Hawks and 40 % – Doves. The trajectory of population evolution was determined by the initial prevalence of each individual strategy.

The replicator equation was applied in the study. It was assumed, however, that the evolutionary process does not occur through the extinction of carriers of inferior genes but through the imitation of behaviours of higher utility. Therefore, it was assumed that replicator dynamics is the basic variant of imitation dynamics. Of course, it is possible to use other selection dynamics, whose essence is imitation. The examples include Maynard Smith's replicator dynamics (1982) and imitative logit (or i-logit) dynamics (Weibull 1997). The same applies to the time over which the changes in the population occur. The replicator equation assumes that it is continuous time. An alternative will be to use a discrete-time version of the replicator dynamics (Thomas 1986).

Finally, there arises a question of fundamental importance. How accurately do the evolutionary game theory models describe processes taking place in complex social networks? First of all, it should be remembered that any models are simplified representations of the studied phenomena. This means that the features considered by the researchers as insignificant are omitted at the initial stages of the research. Although models may become more concrete in time, they will always constitute reduced iterations of reality. Secondly, in social sciences (with the possible exception of economics), there is usually not enough data to build models that can claim to provide approximations with the precision expected from the natural sciences. Therefore, when examining specific historical processes, such as the process of radicalization of political attitudes that precedes revolutions and wars, religious and ethnic antagonisms, or the spread of destructive social norms, it must be taken into account that the models of evolutionary game theory will not show how these processes take place in actuality, but only how they would take place under certain arbitrary assumptions. Finally, evolutionary game theory models have considerable heuristic value, which can help identify further research directions.

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